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Mathematical Reviews

Vol. 22, No. 6B June, 1961 Reviews 5056–5532

PROBABILITY

See also 5081, 5146, 5360.

5056:

Sapogov, N. A. On independent terms of a sum of random variables which is distributed almost normally. Vestnik Leningrad. Univ. 14 (1959), no. 19, 78–105. (Russian. English summary)

Author's summary: "The paper contains a proof for the following result. Let $X_1+X_2=X$, where $X_i=(X_i^1, \cdots, X_i^n)$, $i=1, 2$, are independent random n-dimensional variables; the median of components X_i^k ($k=1, 2, \cdots, n$) equals 0. Denote by $\bar{X}=(\bar{X}^1, \cdots, \bar{X}^n)$ a gaussian n-dimensional variable, for which $M\bar{X}^k=0$ ($1\leq k\leq n$),

$$\min_{|l|=1} D\left(\sum_{i=1}^n l^i\bar{X}^i\right)=\sigma^2>0 \quad \left(|l|^2=\sum_{i=1}^n (l^i)^2\right).$$

Let, for a positive $\varepsilon<1$, $\sup_I |P_X(I)-P_{\bar{X}}(I)|\leq\varepsilon$, where I is a set of points $(x^1, x^2, \cdots, x^n)$, for which $a^i\leq x^i\leq b^i$, $1\leq i\leq n$, and $P_Z(I)=P\,(Z\in I)$. Then there exists such a normally distributed vector $\bar{X}_1$ that

$$(*) \quad \sup_I |P_{X_1}(I)-P_{\bar{X}_1}(I)| < C[\ln\ln(1/\varepsilon+2)]^{3n/2-1}\cdot\sigma_1^{-3}(\ln(1/\varepsilon))^{-1/2},$$

where $C=C(n, \sigma)$ is a constant, $\sigma_1^2=\min_{|l|=1} D(\sum_{i=1}^n l^iX_1^{*i})$ and $X_1^*=(X_1^{*1}, \cdots, X_1^{*n})$ is a random vector for which $X_1^{*k}=X_1^k$ if $|X_1^k|\leq N$, $X_1^{*k}=0$ if $|X_1^k|>N$ ($k=1, 2, \cdots, n$; $N=(2\ln(1/\varepsilon))^{1/2}+1$). In case $n=1$, the right part of (*) is to be replaced by $C\sigma_1^{-3}(\ln(1/\varepsilon))^{-1/2}$."

H. P. Edmundson (Pacific Palisades, Calif.)

5057:

Yang, Tsung-pan. Une remarque sur l'espérance conditionnelle. II. Acta Math. Sinica 9 (1959), 330–332. (Chinese)

An extension of an earlier paper in Advancement in Math. 3 (1957), 658–661 [MR 21 #3041].

S. C. Moy (Syracuse, N.Y.)

5058:

Leonov, V. P.; Širyaev, A. N. Sur le calcul des semi-invariants. Teor. Veroyatnost. i Primenen. 4 (1959), 342–355. (Russian. French summary)

Let $\eta=\{\eta_1, \cdots, \eta_k\}$ be a k-dimensional random vector and $\varphi_\eta(\alpha_1, \cdots, \alpha_k)$ its characteristic function; it is known that

$$\varphi_\eta(\alpha_1, \cdots, \alpha_k)=\sum_{\nu_1+\cdots+\nu_k\leq n} \frac{i^{\nu_1+\cdots+\nu_k}}{\nu_1!\cdots\nu_k!} m_\eta^{(\nu_1,\cdots,\nu_k)}\alpha_1^{\nu_1}\cdots\alpha_k^{\nu_k}+o(|\alpha|^n),$$

$$\ln\varphi_\eta(\alpha_1, \cdots, \alpha_k)=\sum_{\nu_1+\cdots+\nu_k\leq n} \frac{i^{\nu_1+\cdots+\nu_k}}{\nu_1!\cdots\nu_k!} s_\eta^{(\nu_1,\cdots,\nu_k)}\alpha_1^{\nu_1}\cdots\alpha_k^{\nu_k}+o(|\alpha|^n),$$

where $|\alpha|=|\alpha_1|+\cdots+|\alpha_k|$ and

$$m_\eta^{(\nu_1,\cdots,\nu_k)}=M\{\eta_1^{\nu_1}\cdots\eta_k^{\nu_k}\},$$

$$s_\eta^{(\nu_1,\cdots,\nu_k)}=\frac{\partial^{\nu_1+\cdots+\nu_k}}{\partial\alpha_1^{\nu_1}\cdots\partial\alpha_k^{\nu_k}}\ln\varphi_\eta(\alpha_1, \cdots, \alpha_k)\big|_{|\alpha|=0}$$

are respectively the moments and the cumulants of η. Let $\xi=\{\xi_{ij}\}$, $1\leq i\leq k$, $1\leq j\leq m_i$, be a random vector, $\xi_{i\cdot}=\{\xi_{i1}, \cdots, \xi_{im_i}\}$, $\nu_{i\cdot}=\{\nu_{i1}, \cdots, \nu_{im_i}\}$. Suppose that

$$\eta_i=Q_i(\xi_{i\cdot})=\sum_{\nu_{i1},\cdots,\nu_{im_i}} a^{(i)}_{\nu_{i1},\cdots,\nu_{im_i}}\xi_{i1}^{\nu_{i1}}\cdots\xi_{im_i}^{\nu_{im_i}} =\sum_{\nu_{i\cdot}} a^{(i)}_{\nu_{i\cdot}}\xi_{i\cdot}^{\nu_{i\cdot}},$$

where $a^{(i)}_{\nu_{i\cdot}}$ are real constants, $a^{(i)}_{\nu_{i\cdot}}\neq 0$ only for $0<|\nu_{i\cdot}|\leq N$. In this paper explicit expressions for the cumulants of η by means of the cumulants of ξ are given; as example the cumulants of a random function depending quadratically upon a Gaussian one are calculated.

R. Theodorescu (Bucharest)

5059:

Karpelevič, F. I.; Tutubalin, V. N.; Šur, M. G. Limit theorems for compositions of distributions in the Lobačevskiĭ plane and space. Teor. Veroyatnost. i Primenen. 4 (1959), 432–436. (Russian. English summary)

A Lobačevskiĭ plane [space] L can be represented as the interior of the unit circle [sphere], the motions being the projective transformations of the circle [sphere] onto itself. Let θ_x denote a motion in L which carries the origin into the point x. For two Borel measures μ_1 and μ_2 on L which are symmetric (that is, invariant under rotations around the origin) let the convolution $\mu_1*\mu_2$ be defined by $\mu_1*\mu_2(\Gamma)=\int_L \mu_1(\theta_x^{-1}\Gamma)\mu_2(dx)$. A normal distribution on L and the characteristic function of a symmetric probability measure on L are defined. Theorems on characteristic functions of convolutions and a central limit theorem with conditions analogous to Lindeberg's are stated without proofs. Applications in physics are mentioned.

W. Hoeffding (Chapel Hill, N.C.)

5060:

Robinson, Enders A. ★An introduction to infinitely many variates. Griffin's Statistical Monographs & Courses, No. 6. Hafner Publishing Co., New York, 1959. 132 pp. $4.75.

This fact-filled and well-written book aims to give "a

concise presentation of probability theory, limit theorems, and stationary stochastic processes .... So that overall unity would not be lost in details and lengthy description, many of the proofs and applications are either outlined or left as exercises. Throughout the book the student is encouraged to go to the references, which is the key toward making him research-minded."

The basic notions of measure theory and probability theory are presented in Chapters 1 and 2 (29 pages). The central limit problem (conditions for the convergence in law of a sequence of sums of independent random variables) and infinitely divisible laws are treated in Chapter 3 (15 pages). Chapters 4–6 (35 pages) are a clear self-contained treatment of linear operators in Hilbert space and their spectral representations.

Chapter 7 (30 pages) treats (wide sense) stationary stochastic processes as families of elements in a Hilbert space. The author gives a thorough treatment of the spectral decomposition theorem and the Wold decomposition theorem. One notion of which the author makes good use and which is not even mentioned in most books on stationary processes is that of subordination, introduced by Kolmogorov in 1941; if $x_1(t)$ and $x_2(t)$, defined for $t=0, \pm 1, \cdots$, are a multiple stationary time series, then $x_2(t)$ is subordinate to $x_1(t)$ if the Hilbert space spanned by $\{x_2(t), t=0, \pm 1, \cdots\}$ is contained in the Hilbert space spanned by $\{x_1(t), t=0, \pm 1, \cdots\}$. References are given to work on statistical inference on time series; in addition, the last 5 pages of the text discusses the author's own work on wavelet theory. Fifteen pages of exercises, 3 pages of references, and a 4-page index conclude the book. *E. Parzen* (Stanford, Calif.)

5061:

Woll, John W., Jr. Homogeneous stochastic processes. Pacific J. Math. **9** (1959), 293–325.

Consider a homogeneous space (X, G) with homogeneous transition probabilities $P(x, A)$, i.e., $P(x, A)$ is a probability measure for fixed x, measurable in x for fixed A and invariant with respect to G: $P(x, A) = P(g(x), g(A))$, $g \in G$. A homogeneous process is defined by homogeneous transition probabilities $P(t, x, dy)$. The author studies such processes by considering the associated strongly continuous semigroup of operators $T_tf = P(t, \cdot, f)$ mapping the Banach space spanned by constants and continuous functions with compact support into itself. Results are given in terms of the infinitesimal operator of T_t. For a separable version of the homogeneous process the path-functions are investigated qualitatively. It is shown that they have right and left limits and are bounded on bounded t-intervals. If P is a homogeneous transition probability the homogeneous process associated with the semigroup $T_t = \exp\{\operatorname{tr}(P-1)\}$ is called a compound Poisson process. The author shows that every homogeneous process can be considered as the limit, in a certain sense, of compound Poisson processes. The work of S. Bochner on subordination of stochastic processes [*Harmonic analysis and the theory of probability*, Univ. of Calif. Press, Berkeley and Los Angeles, 1955; MR **17**, 273] is generalized and extended. The case of a commutative group is given particular attention by using Fourier analysis via the group characters.
U. Grenander (Stockholm)

5062:

Onoyama, Takuji. Note on random distributions. Mem. Fac. Sci. Kyushu Univ. Ser. A. **13** (1959), 208–213.

Ito [Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math. **28** (1954), 209–223; MR **16**, 378] and Gel'fand [Dokl. Akad. Nauk SSSR **100** (1955), 853–856; MR **16**, 938] have defined a generalized stochastic process following the ideas of Schwartz distribution theory as a continuous linear functional $X(\phi)$ from the space D of C^∞ functions on the line with compact support to random variables in L_2 of a probability measure space. The author replaces D by K_p, the space of functions on the line satisfying (together with all derivatives) the condition $\sup[|\phi(x)| \exp(c|x|^p)] < \infty$, where c depends on ϕ. He shows that if the given process is stationary in the wide sense, it can be represented in the form $X(\phi) = \int \tilde{\phi}(\lambda) dM(\lambda)$, where: $\tilde{\phi}$ is the Fourier transform of ϕ; M is a random measure of linear Borel sets, satisfying $E\{M(A_1A_2)\} = \mu(A_1A_2)$; μ is a positive measure of linear Borel sets satisfying $\int \exp(-c|\lambda|^p) d\mu(\lambda) < \infty$, for some positive constant c. In the Ito-Gel'fand treatment, μ has the stronger condition that

$$\int (1+\lambda^2)^{-k} d\mu(\lambda) < \infty$$

for some positive k. *J. L. Doob* (Urbana, Ill.)

5063:

Lukaševič, Yu. A remark on a theorem of Hinčin from the theory of stochastic flows. Colloq. Math. **7** (1959/60), 285–287. (Russian)

Let $\psi_r(u)$ be the probability that in a time interval of length u at least r events occur. It is supposed that the events occur in a Palm stream (stationary, ordinary, bounded aftereffect); see Hinčin, Trudy Mat. Inst. Steklov. **49** (1955) [MR **17**, 276]. The author proves that if a_r is the supremum of the zeros of ψ_r then

$$\lim_{u \downarrow a_r} \psi_{r+1}/\psi_r = 0.$$

This result can be used to validate an argument in the above reference which otherwise requires the hypothesis that ψ_r does not vanish near the origin.
J. L. Doob (Urbana, Ill.)

5064:

Steinhaus, H.; Urbanik, K. Poissonsche Folgen. Math. Z. **72** (1959/60), 127–145.

A monotone non-decreasing sequence $\{s_n, n \geq 1\}$ is a Poisson sequence if for every natural number k, disjoint intervals $I_1, \cdots, I_k$, and arbitrary nonnegative integers $m_1, \cdots, m_k$, the relative Lebesgue measure on $[0, \infty)$ of the set of values of t for which $t + I_r$ contains exactly m_r terms of the sequence, for all $r \leq k$, is given by the Poisson distribution formula $\prod_1^k (|I_r|^{m_r}/m_r!) \exp(-|I_r|)$. The sequence is 'Poisson in itself' if the set of integers n for which $s_n + I_r$ contains exactly m_r terms of the sequence for all $r \leq k$ has relative frequency given by the same formula. Neither of the two properties just defined implies the other. The interrelations between sequences of these types, sequences equidistributed on $[0, 1)$, and Poisson stochastic processes are investigated. For example, it is shown that a sequence $\{a_n, n \geq 1\}$ is equidistributed in the strong sense that for every k the points $\{(a_{n+1}, \cdots, a_{n+k}), n \geq 1\}$ are equidistributed on the k-dimensional unit cube,

if and only if the sequence defined by $s_n = -\log \prod_1^n (1-a_j)$ is Poisson in itself. *J. L. Doob* (Urbana, Ill.)

5065:

Šahov, Yu. N. Imitation of simplest Markov processes. Izv. Akad. Nauk SSSR. Ser. Mat. **23** (1959), 815–822. (Russian)

For a Markov chain with finitely many states and strictly positive rational stationary transition probabilities it is shown that one can imitate the (long but finite) sample sequences in a suitable sense by normal periodic systems as defined by Korobov [same Izv. **16** (1952), 212–216; MR **14**, 144] if and only if the absolute probabilities are stationary. It is then shown how to imitate infinite sample sequences using normal periodic systems. The construction is different from that of Postnikov and Pyateckiĭ [ibid. **21** (1957), 729–746; MR **21** #664]. *J. L. Doob* (Urbana, Ill.)

5066:

Volkonskiĭ, V. A. Additive functionals of Markov processes. Dokl. Akad. Nauk SSSR **127** (1959), 735–738. (Russian)

The author announces several results concerning the additive functionals which are the negative logarithms of the multiplicative functionals of Dynkin, each corresponding to a subprocess of a homogeneous Markov process. *K. L. Chung* (Syracuse, N.Y.)

5067:

Prapordžesku, N. [Praporgescu, N.]. On temporally variable probabilities forming a chain. Rev. Math. Pures Appl. **4** (1959), 403–423. (Russian)

This paper is the sequel of a previous one published by the author in Acad. R. P. Romîne. Stud. Cerc. Mat. **9** (1958), 439–480 [MR **21** #2302]. Non-stationary simple Markov chains with a discrete as well as a continuous set of states are investigated. By means of similar methods as used in the quoted paper, expressions for the solutions of the corresponding Chapman-Kolmogorov equations are given. These expressions are used in order to characterise the asymptotic behaviour. It is stated by the author that the results hold true for multiple chains.

R. Theodorescu (Bucharest)

5068:

Girsanov, I. V. Über eine Eigenschaft des nichtsingulären Diffusionsprozesses. Teor. Veroyatnost. i Primenen. **4** (1959), 355–361. (Russian. German summary)

Let $x_t = x_t(\omega)$ be a temporally homogeneous strong Markov process with continuous paths in a measurable topological space. Let P_x be the probability measure if $x_0 = x$ and $P(t, x, \Gamma) = P_x\{x_t \in \Gamma\}$. Let U be an open set with boundary $\gamma(U)$ and let τ_U be the first passage time to γ. Let $\Omega_U = \{\omega : x_t(\omega) \in \gamma(U)$ for some $t\}$, $m_U(x) = \int_{\Omega_U} \tau_U dP_x$, $\pi_U(x, \Gamma) = P_x\{x_{\tau_U} \in \Gamma\}$. The author treats processes in n-dimensional Euclidean space R^n, restricting treatment to "Feller" processes for which the transformation $T_t f(x) = \int f(y) P(t, x, dy)$ changes bounded continuous functions into bounded continuous functions. Let A be the infinitesimal generator of T_t. Let $\pi_V f(x) = \int_{R^n} \pi_V(x, dy) f(y)$. Dynkin [Teor. Veroyatnost. i Primenen. **1** (1956), 38–60; MR **19**, 691] showed that the operator $\mathfrak{A}$ defined by $\mathfrak{A} f(x) = \lim_{V \downarrow x} [\pi_V f(x) - f(x)]/m_V(x)$, where the regions V close down on x, is an extension of A. Thus knowledge of $\pi_V(x, \Gamma)$ and $m_V(x)$ for all x and Γ and a sufficiently wide class of regions V determines the operator A. The purpose of the article is to show that for certain processes knowledge of $\pi_U(x, \Gamma)$ and $m_U(x)$ for a single region U and all x and Γ determines $\mathfrak{A}$ for all x in U. The processes considered are of the diffusion type, where

$$\mathfrak{A} f(x) = \sum_{i,j=1}^{n} b_{ij}(x) \frac{\partial^2 f(x)}{\partial x_i \partial x_j} + \sum_i a_i(x) \frac{\partial f(x)}{\partial x_i} + c(x) f(x), \quad c(x) \leq 0.$$

It is shown that if x_t is a diffusion process with operator $\mathfrak{A}$, and U is a suitably regular region in R^n, then any strong Markov process x_t' with continuous paths, having the same $\pi_U(x, \Gamma)$ and $m_U(x)$ for all x and Γ, has the same operator $\mathfrak{A}$ in U. The result for one-dimensional processes is a consequence of work of Feller [Ann. of Math. (2) **60** (1954), 417–436; MR **16**, 488] and Dynkin [loc. cit.].

T. E. Harris (Santa Monica, Calif.)

5069:

Has'minskiĭ, R. Z. On positive solutions of the equation $\mathfrak{A}u + V = \cdot u = 0$. Teor. Veroyatnost. i Primenen. **4** (1959), 332–341. (Russian. English summary)

The notation and terminology used in this paper is that of E. B. Dynkin. Let $X = (X_t, \tau, M_t, P_x, \theta_t)$ be a strong Markov process with domain D and boundary Γ in a metric space (E, ρ); and let $\mathfrak{A}$ be the extended infinitesimal operator (in the sense of Dynkin) associated with X. The main result is as follows: Let $\{X_t\}$ be a strong Feller process with regular boundary Γ. Then

$$M_x \exp\left\{\int_0^\tau V(X_t) dt\right\} < +\infty$$

(where $V(x)$ is a nonnegative continuous function in D) if and only if the equation (*) $\mathfrak{A}u + Vu = 0$, $u \in D(\mathfrak{A})$, has a solution which is positive and continuous in $D \cup \Gamma$. This theorem is then used to study the existence of a unique solution of (*) when $\mathfrak{A}$ is the elliptic operator

$$L = \sum_{i,j=1}^{n} A_{ij} \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{n} B_i \frac{\partial}{\partial x_i}.$$

A. T. Bharucha-Reid (Eugene, Ore.)

5070:

Kendall, David G. Hyperstonian spaces associated with Markov chains. Proc. London Math. Soc. (3) **10** (1960), 67–87.

Consider a denumerable Markov chain with state space E $(=\{1, 2, \cdots\})$, and boundary $\mathscr{B}$ [cf. W. Feller, Trans. Amer. Math. Soc. **83** (1956), 19–54; MR **19**, 892]. This paper presents a detailed study of the topological character of the Feller boundary $\mathscr{B}$; for example, it is shown that $\mathscr{B}$ is hyperstonian and of countable type. These results are then used to generalise a theorem of D. Blackwell [Ann. Math. Statist. **26** (1955), 654–658; MR **17**, 754] on the stochastic motion of the process.

A. T. Bharucha-Reid (Eugene, Ore.)

5071:

Karlin, Samuel; McGregor, James. Coincidence properties of birth and death processes. Pacific J. Math. **9** (1959), 1109–1140.

The authors continue their series of papers on birth and death processes with transition probabilities $P_{ij}(t)$, assuming in this paper that the birth and death rates λ_i and μ_i uniquely determine the process and that $\mu_0=0$. As shown in Trans. Amer. Math. Soc. **85** (1957), 489–546 [MR **19**, 989] the determinants:

$$(*)\qquad P\left(t,\begin{matrix}i_1\cdots i_n\\ j_1\cdots j_n\end{matrix}\right)=\det[P_{i_\mu,j_\nu}(t)]$$

$$(i_1<\cdots<i_n,\ \ j_1<\cdots<j_n)$$

are strictly positive when $t>0$, and as shown in Proc. Nat. Acad. Sci. U.S.A. **45** (1959), 375–379 [MR **21** #923] a continuous-time, discrete-space Markov process for which (*) is positive for $t>0$ is a birth and death process. In the paper reviewed below the determinant in (*) is shown to be the probability that n labelled particles starting in states $i_1,\cdots,i_n$, executing the process independently, will be in states $j_1,\cdots,j_n$ at time t without any two of them ever having been coincident (simultaneously in the same state) in the intervening time. Here the authors study the structure of the Markov process describing the transitions of n particles conditioned that no coincidence takes place. The principal tool is an integral representation for (*) derived from that for the transition probabilities. The strong ratio ergodic theorem is generalized by proving that the ratio of two determinants of the type (*) with the same n tends to a finite positive limit as $t\to\infty$. The second problem, of determining for what processes coincidence in a finite state is certain to occur, is completely solved, necessary and sufficient conditions being given in terms of the birth and death rates (the non-trivial case is that of the transient process). A technique for computing the distribution of the time until coincidence is developed and applied to the telephone trunking model and some linear growth models. {The first five pages of this paper should be read in the following order: 1109, 1112, 1111, 1110, 1113.}

F. L. Spitzer (Minneapolis, Minn.)

5072:

Karlin, Samuel; McGregor, James. Coincidence probabilities. Pacific J. Math. **9** (1959), 1141–1164.

The probability interpretation of the determinants (*) in #5071 is derived as a special case of a much more general result concerning a stationary strong Markov process $x(t)$ with transition probabilities $P(t,x,E)$, whose state space is a metric space and whose path functions are continuous on the right: Suppose that n labelled particles start at $x_1,\cdots,x_n$ and execute the process independently. For each permutation σ of $1,2,\cdots,n$ let A_σ denote the event that at time t the particles are in the Borel sets $E_{\sigma_1},\cdots,E_{\sigma_n}$ respectively, without any two of them ever having been coincident in the intervening time. Then

$$\det[P(t,x_\mu,E_\nu)]=\sum_\sigma(\operatorname{sign}\sigma)\Pr(A_\sigma)$$

$$(\mu,\nu=1,2,\cdots,n),$$

the sum running over all σ, with sign $\sigma=1$ for even permutations and -1 for odd ones.

The proof is based on the reflection principle, made rigorous by appealing to the strong Markov property. Sufficient conditions are given for the total positivity of $P(t,x,E)$, for processes on the line, i.e., that (*) is positive for all $t>0$, when $x_1<\cdots<x_n$ and $E_1<\cdots<E_n$. This is the case when $P(t,x,E)>0$ for all $t>0$, E any open set, and the process has continuous path functions. A partial converse is given, based on a restriction about the local character of $P(t,x,E)$. The most general one-dimensional spatially homogeneous (infinitely divisible) process whose $P(t,x,E)$ is totally positive is shown to be Brownian motion with drift. *F. L. Spitzer* (Minneapolis, Minn.)

5073:

Rosenblatt, M. Stationary processes as shifts of functions of independent random variables. J. Math. Mech. **8** (1959), 665–681.

The following two problems in the structure theory of strictly stationary stochastic processes are investigated. Let $\{x_n, n=0,\pm1,\cdots\}$ be a stationary process with T the corresponding one-step shift operator. Let B_n be the Borel field of sets generated by $x_n, x_{n-1},\cdots$.

Problem 1: (a) When is it possible to find a random variable ξ_0 measurable with respect to B_0, independent of $x_{-1},x_{-2},\cdots$, and such that $B_0=B_{-1}\times A_0$ (product field) where A_0 is the Borel field generated by ξ_0? (b) If (a) is satisfied, the random variables $\xi_n=T^n\xi_0$ $(n=0,\pm1,\cdots)$ are independent and identically distributed. Let A_n be the Borel field generated by ξ_n. Is x_n measurable with respect to $\cdots\times A_{n-1}\times A_n$?

Problem 2: Can one define a sequence of independent, uniformly distributed (on [0, 1]) random variables ξ_n $(n=0,\pm1,\cdots)$ and a Borel function $g(\xi)=g(\xi_0,\xi_1,\cdots)$, $\xi=(\cdots,\xi_{-1},\xi_0,\xi_1,\cdots)$, such that $y_n=g(T^n\xi)$ $(n=0,\pm1,\pm2,\cdots)$ has the same probability structure as $\{x_n\}$? Here T is the shift operator $T\xi=T(\cdots,\xi_{-1},\xi_0,\xi_1,\cdots)=(\cdots,\xi_0,\xi_1,\xi_2,\cdots)$.

First, it is shown that a necessary condition for problem 2 (and hence also for problem 1) to hold is that the process be "purely non-deterministic", i.e., that the only functions measurable with respect to the Borel field $\bigcap_n B_n$ be the constant functions. The central results of the paper are obtained for stationary, purely non-deterministic Markov chains: problem 1 is studied for stationary, finite state Markov chains satisfying an additional property which the author calls uniformity. For such Markov chains the author derives a necessary and sufficient condition for the first problem to have a positive answer. This condition is algebraic in character, being stated in terms of a property of the semigroup generated by a family of matrices related to the process.

If $\{x_n\}$ is a stationary, purely non-deterministic Markov chain, it is shown that problem 2 has a solution provided there exists a positive ε and a set M of a finite number of states of the chain such that $\sum_{j\in M}p_{ij}>\varepsilon$ for all i.

A sufficient condition for a positive answer to both problems is obtained in the case of a general strictly stationary process. The statement of this condition is omitted here for the sake of brevity, but it is one which is both necessary and sufficient in order that

$$\lim_{k\to\infty}E|x_n-E(x_n|\,\xi_n,\cdots,\xi_{n-k})|=0.$$

Using this result, the author gives an example of a strictly stationary Markov process $\{x_n\}$ for which problem 1 has a positive answer but for which the random variables ξ_n cannot be taken to be of the form $F(x_n|x_{n-1},x_{n-2},\cdots)$ (conditional distribution function), where $F(a_n|a_{n-1},\cdots)$

is a continuous, strictly increasing function of a_n. This example seems to necessitate a modification of a conjecture made by N. Wiener and the reviewer in some unpublished work. *G. Kallianpur* (Bloomington, Ind.)

5074:

Masani, P.; Wiener, N. On bivariate stationary processes and the factorization of matrix-valued functions. Teor. Veroyatnost. i Primenen. **4** (1959), 322–331. (Russian summary)

Let $F:[f_{ij}]$ be a 2×2 non-zero non-negative hermitian matrix-valued function with entries in L_1 on the unit circle, such that $\det F=0$ almost everywhere on the circle. Then $F=\Phi\Phi^*$ almost everywhere, for some Φ whose entries are in L_2 and have vanishing nth Fourier coefficients for $n<0$, if and only if, for $i=1$ or 2, $\log f_{ii}\in L_1$ and, for $j\neq i$, f_{ji}/f_{ii} is the radial limit of the quotient of a pair of functions regular and in the Hardy class H_δ on the unit disc, for some $\delta>0$. This result is applied to the case when F is the derivative of the spectral distribution function of a bivariate stationary process to find a necessary and sufficient condition that the process be regular from the point of view of prediction theory.
J. L. Doob (Urbana, Ill.)

5075:

Girsanov, I. V. Spectra of dynamical systems generated by stationary Gaussian processes. Dokl. Akad. Nauk SSSR (N.S.) **119** (1958), 851–853. (Russian)

Let $x(t,w)=\int e^{it\lambda}\phi(d\lambda,w)$ denote a real stationary (in the narrow sense) process. Let $F(d\lambda)=M(|\phi(d\lambda,w|^2)$ be its spectral measure. The transformation $S_\tau:S_\tau(x(t,w))=x(t+\tau,w)$ preserves the measure in the space Y of trajectories of the process. Let U^τ be the unitary operator, corresponding to S_τ, defined in the Hilbert space of real functions on Y. The author studies the spectrum of U^τ for processes with a continuous spectral measure $F(d\lambda)$. Let $\nu(\lambda)$ be the multiplicity function and ρ the maximal type of the spectrum of U^τ. If $\nu(\lambda)\equiv 1$, then the spectrum is called simple. If there exists a set A such that $\rho(A)>0$ and $\nu(\lambda)=1$ if $\lambda\in A$, then the spectrum is said to have a simple component. The author constructs (among others) examples of (a) a stationary Gaussian process with a simple continuous spectrum (the only known examples of continuous spectra up to the present have been of multiplicity $\aleph_0$), and (b) a process with a non-homogeneous spectrum where $\nu(\lambda)$ takes only the values 1 and $\aleph_0$.

Conditions are given for a process to have a simple component, and the following theorem is stated: If $\nu(\lambda)$ is the multiplicity function of a process with a continuous $F(d\lambda)$, then either $\nu(\lambda)\equiv 1$ or it increases unboundedly. No proofs are given. A theorem of K. Ito is quoted as being at the basis of this work. *Y. N. Dowker* (London)

5076:

Rozanov, Yu. A. Spectral theory of n-dimensional stationary stochastic processes with discrete time. Uspehi Mat. Nauk **13** (1958), no. 2 (80), 93–142. (Russian)

A survey article constituting a systematic presentation of the elements of the spectral theory of n-dimensional stationary stochastic processes with discrete time and the problem of extrapolation and interpolation for such processes. The author uses the "coordinate-free" concept of stationary stochastic process proposed by A. N. Kolmogorov, allowing many of the results to be carried over automatically to infinite-dimensional processes; namely, a stationary process $\{A, x_t(a)\}$ is the pair of a unitary space A and an isometric operator $x_t(a)$ from A to a hilbert space H of random variables, such that, for any $a\in A$, $Mx_{t+\tau}(a)\bar{x}_t(a)$ is independent of t.

In chapter I the spectral decomposition of stationary processes is established and the notions of spectral and correlation functions and spectral density introduced. Chapter II contains many facts of an auxiliary nature, very important for the subsequent development. In particular it gives the representation of a process with absolutely continuous spectral function as a moving sum of a sequence of uncorrelated variables. Chapter III is devoted to the problem of linear extrapolation. The notions of regular and singular process are introduced in the natural way in generalization of the corresponding notions in the theory of one-dimensional stationary processes [Doob, *Stochastic processes*, Wiley, New York, 1953; MR **15**, 445; chapter XII]. Various criteria are established for the regularity of a process; of these, some were published by V. N. Zasuhin [Dokl. Akad. Nauk SSSR **33** (1941), 435–437; MR **5**, 102], and some were known to M. G. Kreĭn and were presented by him in lectures. Kreĭn also eliminated some flaws in Zasuhin's proofs. There are given as corollaries results which were obtained in the one-dimensional case by A. N. Kolmogorov [Byull. Moskov. Gosudarstv. Univ. Mat. **2** (1941), no. 6; MR **5**, 101].

In chapter IV are presented the author's previously published results [Dokl. Akad. Nauk SSSR **116** (1957), 923–926; MR **20** #2032] on interpolation of vector processes, generalizing the corresponding results of A. N. Kolmogorov [Izv. Akad. Nauk SSSR, Ser. Mat. **5** (1941), 3–14; MR **3**, 4] and A. M. Yaglom [Uspehi Mat. Nauk **4** (1949), no. 4, 173–178; MR **11**, 119].

{Reviewer's remarks: The problem of extrapolation of n-dimensional stationary processes is also treated in the article of Wiener and Masani, Acta Math. **98** (1957), 111–150 [MR **20** #4323].}
M. I. Yadrenko (RŽMat **1959** #10208)

5077:

Karpeev, G. A. On a class of non-stationary random functions reducing to stationary ones. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 6, 106–111. (Russian)

Motivated by the statistical needs of research in radio wave propagation in the iono- and tropospheres, the author investigates a simple class of non-stationary stochastic processes having the form $X(t)=\varphi(t)+x(t)$, where $\varphi(t)$ is an ordinary function which may be odd, even, or linear and $x(t)$ is a stationary stochastic process with first order normal distribution. The author calculates the first order distribution of $\xi(t)=X(t)\pm X(-t)$ by standard methods. Finally, he investigates the relation between moments of $\xi(t)$ and their estimators.
H. P. Kramer (Santa Barbara, Calif.)

5078:

Moran, P. A. P. ★The theory of storage. Methuen's Monographs on Applied Probability and Statistics.

Methuen & Co., Ltd., London; Wiley & Sons, Inc., New York; 1959. 111 pp. $2.50.

The author gives a useful connected account of some probability problems that arise in theories of inventories and of dams. Inventories: brief accounts are given of Hammersley's storage model and Segerdahl's theory of insurance risk, followed by a more detailed comparison of two replacement policies based on work of Pitt [J. London Math. Soc. **21** (1946), 16–22; MR **8**, 281] and Gani [Biometrika **42** (1955), 179–200; MR **17**, 1097], and reference to more complex models. Dams: the account is based on the author's pioneer work on finite and infinite dam models, and developments by Gani, Kendall and other speakers at a Royal Statistical Society symposium [J. Roy. Statist. Soc., Ser. B, **19** (1957), 181–233; MR **19**, 1092], with mention of further work not published at the time of writing (1958). The author draws attention to the usefulness, but also to the inherent limitations, of these storage models. He concludes with chapters on Monte Carlo, and other, statistical methods, and on the programming of storage systems. The mathematical prerequisites are introduced or summarised in the opening chapter. Here, and perhaps also elsewhere, subheadings seem desirable. *J. L. Mott* (Edinburgh)

5079:

Давенпорт, В. Б. [Davenport, Wilbur B., Jr.]; Рут, В. Л. [Root, William L.]. ★Введение в теорию случайных сигналов и шумов. [An introduction to the theory of random signals and noise]. Translated from the English by B. G. Belkin; edited by R. L. Dobrušin. Izdat. Inostr. Lit., Moscow, 1960. 468 pp. 20.25 r.

For original [McGraw-Hill, New York, 1958] see MR **19**, 1090.

5080:

Li, Hen Von. On the theory of optimal filtration of a signal in the presence of internal random noise. Teor. Veroyatnost. i Primenen. **4** (1959), 458–464. (Russian. English summary)

The following problem is considered. We have a linear, constant-coefficient network, consisting of a number of blocks in series, whose transfer functions are given, and also a feedback loop containing a block with a transfer function, $W^{(2)}(p)$, $p=d/dt$, to be determined. Noise is injected into positions intermediate to the various blocks. The input is a function $s(t)+n(t)$, where $n(t)$ is a noise, and $s(t)$, the useful pure signal, is of the form $s(t)=\sum_{i=0}^{l} a_i t^i+m(t)$, where $m(t)$ is a stochastic process, and $\{a_i\}$ are unknown. The desired output is $s^*(t)=H(p)s(t)$, $H(p)$ given. The problem is to determine $W^{(2)}(p)$ so that the actual output, $x(t)$, is an unbiased least-squares estimate of $s^*(t)$. This is solved under the hypothesis that the various stochastic processes are independent stationary ones, with given correlation functions, by reduction to an integral equation of the form $\int_0^T K(\tau)R(t-\tau)d\tau=f(t)$ (T=observation time). The solution, if certain spectral functions are rational, can, as in Dolph and Woodbury, Trans. Amer. Math. Soc. **72** (1952), 519–550 [MR **14**, 295], be expressed in terms of Green's functions of associated differential operators. *E. Reich* (Aarhus)

STATISTICS

See also 5080.

5081:

Brunk, H. D. ★An introduction to mathematical statistics. Ginn and Co., Boston, Mass., 1960. xi+403 pp. $7.00.

This book is intended for a one- or two-semester course with calculus prerequisite. Part one (Probability) has chapters on elementary probability spaces, general probability spaces, random variables, combined random variables, and the algebra of expectations. Part two (Statistics) has chapters on random sampling, the weak law of large numbers, estimation, the central limit theorem, confidence intervals and hypothesis testing, decision theory, regression, sampling normal populations, more testing, experimental design and analysis of variance, other sampling methods, and distribution-free methods. The later chapters are largely independent of one another. There are 35 pages of tables.

The exposition is clear and, especially in Part one, usually careful mathematically for a book at this level; for instance, the sample space for a sample of two is set up in detail. Principal criticisms: the introduction of the mean fails to mention that it is a long-run average (just as a probability is a long-run proportion). The definition of independence is rather round-about (and still omits the case that one variable has a density while the other is discrete). Unusual definitions of Bayes estimate and efficiency are given. Distributions are often obtained by moment generating functions; practice in direct derivation seems more useful pedagogically. Ties are broken at random (pp. 344, 357–358), which is never good statistics, and here (in connection with runs tests) does not even simplify the mathematics.

The proofs and derivations are good. Theorems are quoted without proof when appropriate. A few derivations are unnecessarily elaborate (in particular, pp. 198, 203–204, 303, 304, 334–335, 345, 347–349). There are very few outright errors.

The real short-coming of this book is shared with its competitors: it conceals the basic philosophical issues of inference from samples. Even the chapter on "statistical decision theory" doesn't convey the depth or breadth of the fundamental issues. Nowhere does the book discuss why one should (or should not) have confidence in a particular confidence statement or what inferential meaning (if any) a test of hypothesis has. It gives none of the direct arguments for basing inferences on likelihoods, nor the absurdities one can obtain by the standard methods because they use tail probabilities. It doesn't discuss the arbitrariness of standard methods (choice of statistics and level) but it dismisses Bayes procedures because of the arbitrariness of the prior (which doesn't dispose of the fact that only Bayes procedures satisfy some inescapable rules of consistency).

The book is an excellent introduction to the mathematics of probability and an adequate introduction to the mathematics of standard statistics. The reviewer knows of no book introducing this mathematics and the theoretical problems of inference at the same time. As a result, unfortunately, our students are realizing late, or never, that such problems exist.

J. W. Pratt (Cambridge, Mass.)

5082:

Salvemini, Tommaso. Transvariazione tra medie di "k" valori di due variabili statistiche. Statistica. Bologna **20** (1960), 135–143.

Let x, y be random variables whose expectations a_x and a_y exist. $T=P(x<y)/P(x-a_x<y-a_y)$ is called a measure of the transvariation relative to a_x and a_y. The author tabulates the values of T for a series of normally distributed independent random variables x and y with a view to applications to the sampling of normal populations. *L. Schmetterer* (Hamburg)

5083:

Obuhov, A. M. On statistically orthogonal expansions of empirical functions. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1960**, 432–439. (Russian)

5084:

Kuiper, Nicolaas H. On the random cumulative frequency function. Nederl. Akad. Wetensch. Proc. Ser. A **63** = Indag. Math. **22** (1960), 32–37.

This is an expanded version of a note by the author which appeared in Ann. Math. Statist. **30** (1959), 251–252 [MR **21** #356]. In both of these papers the author gives an alternative and simpler proof of theorem 3 in the paper by Z. W. Birnbaum and R. Pyke [ibid. **29** (1958), 179–187; MR **20** #393]. See also theorem 1 in M. Dwass, ibid. **29** (1958), 188–191 [MR **20** #2051].

Benjamin Epstein (Palo Alto, Calif.)

5085:

Epstein, Benjamin. Estimation of the parameters of two parameter exponential distributions from censored samples. Technometrics **2** (1960), 403–406.

A simple exposition based on results obtained by B. Epstein and M. Sobel [Ann. Math. Statist. **25** (1954), 373–381; MR **15**, 810]. *D. M. Sandelius* (Åmål)

5086:

Sen, Pranab Kumar. On the estimation of the population size by capture-recapture method. Calcutta Statist. Assoc. Bull. **9** (1960), 93–110.

In a population of size N there are n_1 marked members: the problem of estimating N on the basis of the r marks recovered in a sample of size n has been extensively studied. In this paper detailed series expressions are given for inverse first and second moments of the binomial, Poisson and hypergeometric distributions based as usual on inverse factorial series development. These are then used to derive asymptotic variances for several estimates of N (asymptotic as $n_1(n/N)\to\infty$) which are supposed to be an improvement over those previously given. The author, however, confuses first and second order terms.

He then goes on to give some partial results on a sequential approach to this problem, apparently unaware of the complete treatment due to Goodman [Ann. Math. Statist. **24** (1953), 56–69; MR **14**, 776].

D. G. Chapman (Seattle, Wash.)

5087:

Khatri, C. G. On testing the equality of parameters in k rectangular populations. J. Amer. Statist. Assoc. **55** (1960), 144–147.

Consider k rectangular populations with ranges θ_i $(i=1, 2, \cdots, k)$. The author wishes to test the null hypothesis H_0: $\theta_1=\theta_2=\cdots=\theta_k$. To this end samples of size n are drawn from each of the k populations. If the lower limit of each of the k populations is arbitrary, one computes $u=\min s_i/\max s_i$ (where s_i is the range of the sample drawn from the ith population) and rejects H_0 if the ratio is too small. If the lower limit of each of the k populations is zero, one computes $v=\min d_i/\max d_i$ (where d_i is the largest value observed in the sample drawn from the ith population) and rejects H_0 if the ratio is too small. 5% points of u and v are tabulated for selected values of k and n. Questions of power are not considered. *Benjamin Epstein* (Palo Alto, Calif.)

5088:

Anderson, Edgar. A semigraphical method for the analysis of complex problems. Technometrics **2** (1960), 387–391.

[Also published in Proc. Nat. Acad. Sci. U.S.A. **43** (1957), 923–927.] The author gives a summary of his multivariate plotting method. This is based on plotted circles, with extra variables and their values indicated by rays of varying directions and lengths.

D. M. Sandelius (Åmål)

5089:

Kosambi, D. D. The method of least-squares. J. Indian Soc. Agric. Statist. **11** (1959), 49–57.

In this expository paper the author discusses the method of least squares in an abstract metric space and obtains solutions for a system of linear equations.

R. G. Laha (Calcutta)

5090:

de Heinzelin de Braucourt, Jean. Principes de diagnose numérique en typologie. Acad. Roy. Belg. Cl. Sci. Mém. Coll. in-4° (2) **14** (1960), no. 6, 72 pp.

Empirical search for combinations of characters which have a bimodal distribution. Examples of simple statistical discriminants. *C. A. B. Smith* (London)

5091:

Siotani, Minoru. Notes on multivariate confidence bounds. Ann. Inst. Statist. Math. Tokyo **11** (1960), 167–182.

This paper deals with confidence bounds for contrasts involving the mean vector μ_i of p-variate normal distributions which are $N(\mu_i, \Lambda)$, $i=1, 2, \cdots, k$. The results give specific subsets of the bounds of S. N. Roy and Bose [Ann. Math. Statist. **24** (1953), 513–536; MR **15**, 726]. They include (i) simultaneous confidence bounds relating to a specific set of independent comparisons among the μ_i's, (ii) simultaneous confidence bounds when one of the k experiments is a standard one, and (iii) simultaneous bounds on the contrasts $\mathbf{a}'(\mu_i-\mu_j)$ for all non-null $\mathbf{a}$ and for $i\neq j$. The method of calculating the 100α percent points of the quantities needed in setting these bounds is also discussed. *P. S. Dwyer* (Ann Arbor, Mich.)

5092:

Dudzinski, M. L. Estimation of regression slope from tail regions with special reference to the volume line. Biometrics **16** (1960), 399–407.

Bartlett's modification of Wald's method for estimating the slope of a regression line with both variables subject to error is studied for the case where there is a heteroscedasticity in the model. Comparisons are made with the weighted and unweighted regression estimates for some empirical forestry data. In the Bartlett procedure, the slope is estimated as the join of the means of the upper and lower thirds of the observations. For the empirical data it is shown that this subdivision should be altered for maximum efficiency.

D. G. Chapman (Seattle, Wash.)

5093:

Quandt, Richard E. Tests of the hypothesis that a linear regression system obeys two separate regimes. J. Amer. Statist. Assoc. **55** (1960), 324–330.

In a previous article [same J. **53** (1958), 873–880; MR **20** #6747] the author has given a method of estimating the parameters a_1, b_1, σ_1, a_2, b_2, σ_2 and t of a system

$$y_i = a_1+b_1x_i+u_i \quad (i = 1, \cdots, t)$$
$$y_i = a_2+b_2x_i+v_i \quad (i = t+1, \cdots, T),$$

where the u_i are NID$(0, \sigma_1^2)$ and v_i are NID$(0, \sigma_2^2)$. He also determined the likelihood ratio λ, which he suggested could be used to test the hypothesis that there is only one regression against the alternative that the model has the form described. It is known that under certain regularity conditions $-2 \log \lambda$ is asymptotically distributed as χ^2, but it is also clear that the regularity conditions are not fulfilled here. The author reports on a sampling experiment that indicates $-2 \log \lambda$ does not have a χ^2 distribution with the appropriate d.f. at least for $T=20, 40, 60$. He suggests a number of alternative possible tests, each of which has some problems.

D. G. Chapman (Seattle, Wash.)

5094:

Cox, D. R. Regression analysis when there is prior information about supplementary variables. J. Roy. Statist. Soc. Ser. B **22** (1960), 172–176.

In seeking to measure the regression of a random variable (RV), Y, on an independent variable, x, one may have available an ancillary random variable, U, stochastically related to both Y and x. Several instances of this situation are pointed out and in the special case $Y_i=\alpha+\beta x_i+\varepsilon_i$ (α, β unknown parameters, ε_i uncorrelated RV's) where in fact $Y_i=\theta+\phi U_i+\zeta_i$ and $U_i=\lambda+\mu x_i+\eta_i$ (θ, ϕ, λ, μ unknown parameters, ζ_i and η_i uncorrelated RV's)—so that $\beta=\phi\mu$— it is shown that the estimate of β which is the product of the regression coefficient of U on x and that of Y on U is asymptotically more precise than the direct sample regression coefficient of Y on x.

P. Meier (Chicago, Ill.)

5095:

Steyn, H. S. On regression properties of multivariate probability functions of Pearson's types. Nederl. Akad. Wetensch. Proc. Ser. A. **63**=Indag. Math. **22** (1960), 302–311.

In a previous paper [same Proc. **60** (1957), 119–127; MR **18**, 771], the author gave necessary and sufficient conditions for the regression function of X_1 on $X_2, \cdots, X_n$, to be of the form $Q(X_2, \cdots, X_n)/R(X_2, \cdots, X_n)$, where Q and R are polynomials. This theorem is now applied in order to obtain regression properties of multivariate distributions. In particular, the multivariate normal, Wishart, multivariate beta distributions are considered.

I. Olkin (Minneapolis, Minn.)

5096:

Siegel, Sidney; Tukey, John W. A nonparametric sum of ranks procedure for relative spread in unpaired samples. J. Amer. Statist. Assoc. **55** (1960), 429–445.

A test is proposed for relative spread. Under the null hypothesis, the two populations are identical; in particular, they must have a common mean. The null distribution of the test statistic is the same as the distribution of the Wilcoxon two-sample statistic. This distribution is presented with good tables and approximations. The analysis of the power function is not examined.

I. R. Savage (Cambridge, Mass.)

5097:

Green, Bert F., Jr.; Tukey, John W. Complex analyses of variance: general problems. Psychometrika **25** (1960), 127–152.

Some general problems and principles in the application of the analysis of variance to complex problems are considered. An illustrative example is discussed in detail.

P. Meier (Chicago, Ill.)

5098:

Mandel, John. The analysis of latin squares with a certain type of row-column interaction. Technometrics **1** (1959), 379–387.

A straightforward least squares analysis is presented for a latin square design using the model

$$y_{ijk} = \mu+\rho_i+\gamma_j+\rho A_i B_j+\theta_k+\varepsilon_{ijk},$$

where μ is the over-all mean, ρ_i, γ_j, θ_k the main effects of the three factors, and $\rho A_i B_j$, with the A_i and B_j known, is the form assumed for the interaction of the first two factors.

P. Meier (Chicago, Ill.)

5099:

Bryant, Edward C.; Hartley, H. O.; Jessen, R. J. Design and estimation in two-way stratification. J. Amer. Statist. Assoc. **55** (1960), 105–124.

Methods are discussed for taking advantage of two-way stratification when the sample size is smaller than the total number of strata. A numerical example is provided. Derivation of the more complex formulas is referred to the author's unpublished doctoral dissertation ["An analysis of some two-way stratifications", 1955, Iowa State College Library, Ames, Iowa].

P. Meier (Chicago, Ill.)

5100:

Tomassetti, Alvaro. Un'applicazione dell'analisi sequenziale. Statistica. Bologna **20** (1960), 221–240.

Using the principles of Wald's sequential analysis, the author estimates the mean value and the variance of a statistical field, proceeding not by way of samples taken at random, but according to a rule, namely, in a decreasing

order of the character investigated. The necessary set of samples must satisfy the following inequalities:

$$\left|\frac{\overline{m_e}}{\overline{m_u}}-1\right| \leq K_1 \quad\text{and}\quad \left|\frac{\overline{\sigma_e^2}}{\overline{\sigma_u^2}}-1\right| \leq K_2,$$

where K_1, K_2 are the admissible errors,. $\overline{m_e}$, $\overline{m_u}$ the respective mean value of these samples and of the field, and $\overline{\sigma_e^2}$, $\overline{\sigma_u^2}$ the variance of the same sets. The quantity having the most important role in the investigation is the ratio $\alpha=(f_1-f)/(f_1-f_0)$, where f represents the corresponding characteristic random variable, f_0 and f_1 the minimum and maximum values obtained in the frame of the sample. The author gives the expressions $m(\alpha)$, $\overline{\sigma^2}(\alpha)$ for the mean value and the variance corresponding to the value α, and determines α, and thus $f(\alpha)$ such that the foregoing inequalities are satisfied.

O. Onicescu (Bucharest)

5101:

Ghosh, Jayanta. On some properties of sequential *t*-test. Calcutta Statist. Assoc. Bull. **9** (1960), 77–86.

Suppose given a sequence of independent normal random variables with unknown mean θ and variance σ^2. Let $\omega_0=\{(\theta,\sigma):\theta=\theta_0,\sigma>0\}$, $\omega_1=\{(\theta,\sigma):|\theta-\theta_0|=\delta\sigma,\sigma>0\}$, $\omega_2=\{(\theta,\sigma):|\theta-\theta_0|\geq\delta\sigma,\sigma>0\}$, where $\delta>0$ and θ_0 are specified. Let Λ_i $(i=0,1,2)$ denote the set of all probability measures, λ, on the parameter space which assign probability 1 to ω_i. For $i=1,2$, let $C_i(\alpha,\beta)$ denote the class of all tests of ω_0 against ω_i whose average probability (for some $\lambda\in\Lambda_0$) of choosing ω_i is α and whose average probability (for some $\lambda\in\Lambda_i$) of choosing ω_0 is β; $C_i^*(A,B)$, the class of all sequential probability ratio tests, with constants A, B, of one averaged normal density $(\lambda\in\Lambda_0)$ against another $(\lambda\in\Lambda_i)$. A rule in $C_i(\alpha,\beta)$ $[C_i^*(A,B)]$ is said to be doubly minimax (d.m.) in this class if the sup on ω_0 of the probability it chooses ω_i and the sup on ω_i of the probability it chooses ω_0 are simultaneously minimum in this class. A. Wald [*Sequential analysis*, Wiley, New York, 1947; MR **8**, 593] showed that a sequential t-test in $C_1(\alpha,\beta)$ was d.m. in this class. His extension of this property to $C_2(\alpha,\beta)$ contains an error. Using his approximations to the error probabilities of a sequential probability ratio test, Wald concluded that a sequential t-test in $C_i^*((1-\beta)/\alpha,\beta/(1-\alpha))$ was also (approximately) d.m. in this class. Hoel [Skand. Aktuarietidskr. **37** (1954), 19–22; MR **16**, 604], by counterexample, showed that this latter property could not hold exactly. The paper under review is a discussion of the good properties of the one-sided and two-sided sequential t-tests whose purpose is ostensibly to clarify Wald's results and to show that Hoel's criticism is "wide of the mark" in the sense that, even though "approximate" equality may hold, $C_i^*((1-\beta)/\alpha,\beta/(1-\alpha))\neq C_i(\alpha,\beta)$. Hence, a rule in the left-hand class with average error probabilities α_1, β_1, though not d.m. in this class, may still be d.m. in $C_i(\alpha_1,\beta_1)$. This, in essence, had already been pointed out by L. Weiss in the review of Hoel's paper cited above.

M. Skibinsky (W. Lafayette, Ind.)

5102:

Jílek, Miloš; Likař, Otakar. Tolerance limits of a normal distribution. Apl. Mat. **5** (1960), 239–246. (Czech. Russian and English summaries)

A survey of different types of the so-called statistical tolerance limits in the case of a univariate normal distribution is given. On the one hand, the authors state (1) the tolerance limits forming a region within which with a given probability P at least a specified proportion p of the population lies, and on the other hand, they discuss (2) the tolerance limits constructed so as to form a region covering on the average the proportion p of the population. For both cases, the authors give tables of coefficients for the construction of tolerance limits. An inspection of these tables shows that the size of the coefficients (and hence of the tolerance regions) depends on the amount of information available about the population (e.g., the knowledge of some of the parameters, the size of sample). The values of the coefficients are decreasing with decreasing proportion p and in the case (1) with decreasing "confidence probability P".

J. Janko (Prague)

5103:

Mendenhall, W.; Lehman, E. H., Jr. An approximation to the negative moments of the positive binomial useful in life testing. Technometrics **2** (1960), 227–242.

In this paper the authors wish to find the mean and variance of the maximum likelihood estimate (m.l.e.) of the scale parameter of a Weibull distribution (with known shape parameter), when life testing of the sample is discontinued after a fixed time. They show that the mean and variance of this estimate can be expressed as functions of the first two negative moments of a positive binomial random variable. A discrete random variable R is called positive binomial, if it takes on the integer values $r=1, 2,\cdots,n$ with probabilities $p(r)=b(r;n,p)/(1-B(0;n,p))$. The values of $E(1/R^k)$ are calculated for $k=1,2,3,4$; $p=.05(.05)\,.95$; $n=5, 10, 15, 20, 30, 40$. In addition, an approximate formula is obtained for the negative moments of the positive binomial, thus giving an approximate expression for the mean and variance of the m.l.e. of the scale parameter of the Weibull distribution.

Benjamin Epstein (Palo Alto, Calif.)

5104:

Cox, D. R. The analysis of exponentially distributed life-times with two types of failure. J. Roy. Statist. Soc. Ser. B **21** (1959), 411–421.

A number of alternative probability models are described which may be helpful in analyzing failure distributions when there are two types of failure. The models considered include one recently described by Mendenhall and Hader [Biometrika **45** (1958), 504–520; MR **20** #6762]. Some statistical techniques which may be useful in testing whether various models may or may not be suitable are illustrated on an example given in the reference cited above.

Benjamin Epstein (Palo Alto, Calif.)

5105:

Hannan, E. J. ★Time series analysis. Methuen's Monographs on Applied Probability and Statistics. Methuen & Co., Ltd., London; John Wiley & Sons, Inc., New York; 1960. viii+152 pp. $3.50.

This is an introduction to statistical analysis of stationary stochastic processes with discrete time. It reviews the main ideas and methods put forward in this field in recent

years. The theoretical discussion is accompanied by practical considerations and numerical illustrations with a great deal of emphasis on the applications.

After an introductory chapter defining basic notions like the spectral representation and optimal prediction the author describes some statistical techniques proposed for the finite parameter schemes such as the Mann-Wald treatment of autoregressive schemes. The periodogram is studied and its inconsistency is exhibited. Consistent estimates of the spectral density are introduced and their sampling properties are discussed. The author examines tests for various hypotheses, especially for independence and for autoregressive schemes. Here much attention is given to practically useful approximations of the sampling distributions. The same is true for the goodness-of-fit tests. A classical problem that has attracted much attention is removal of a trend from the observed values of the process. The possible ways of doing this are discussed from various points of view and practical conclusions are drawn.

The exposition is clear, sometimes concise, leaving out more difficult derivations or using heuristic arguments, referring the reader to the statistical literature for complete discussions. The didactic skill of the author has made it a very readable book. The prerequisites are modest: no knowledge of time series analysis is assumed, only a reasonably good general background in mathematical statistics. Such a reader will find this book very useful, giving him a good idea of modern time series analysis.

U. Grenander (Stockholm)

5106:

Ogawara, Masami. Time series analysis and stochastic prediction. I, II. Bull. Math. Statist. **8**, 8–53 (1958); 55–72 (1959).

The bibliography of this paper lists 23 papers published by the author in the period 1948–1957. The paper under review is a summary of the theory of time series analysis developed and applied in the author's earlier papers.

According to the author, since only small samples are available of the time series which arise in practice, large sample statistical theory is inadequate. It is better to use an exact theory, valid for small samples, on the assumption that the observed time series obeys a finite parameter scheme of specified form. In Chapters I and II (25 pages), the author gives a thorough review of the theory of discrete and continuous parameters stationary time series with rational spectral density functions. In Chapter III (20 pages), he develops an exact theory of statistical inference based on the fact that if $\{x(t), t=1, 2, \cdots, N\}$ is a sample of a normal stationary time series which is an autoregressive scheme, or multiple Markov process of order h, then given the observations $x(1), \cdots, x(h), x(h+2), \cdots, x(2h), x(2(h+1)+1), \cdots, x(3h), \cdots, x((n-1)(h+1)+1), \cdots, x(nh)$, the conditional distribution of $x(h+1), x(2(h+1)), \cdots, x(n(h+1))$ is that of n independent random variables with a joint probability density function which may be expressed in terms of the means, variances and correlation coefficients of the observations. Chapters IV (Stochastic prediction, 11 pages) and V (Fiducial prediction, 7 pages) use similar observations in treating the problem of predicting the value of a time series.

E. Parzen (Stanford, Calif.)

5107:

Durbin, J. Efficient estimation of parameters in moving-average models. Biometrika **46** (1959), 306–316.

Estimation of the parameters of a finite moving average process is difficult since the maximum likelihood equations are intractable. The author approximates the process by an appropriate autoregressive scheme and estimates the parameters of the latter. In this way estimates of high precision are obtained for the parameters of the original process. They can be used for the construction of large sample tests. Some numerical results are given.

In an appendix the author evaluates the limiting covariance determinant of a second order moving average process. It should be remarked that this is a special case of a general theorem in the theory of Toeplitz forms; see U. Grenander and G. Szegö, *Toeplitz forms and their applications* [Univ. of California Press, Berkeley, Calif., 1958; MR **20** #1349; p. 76]. *U. Grenander* (Stockholm)

5108:

Walker, A. M. Some consequences of superimposed error in time series analysis. Biometrika **47** (1960), 33–43.

Let $x_1, x_2, \cdots, x_n$ be an observed sample of a stationary stochastic process $x_t=u_t+\eta_t$, where u_t is an autoregressive process of order p and η_t is pure white noise independent of u_t. From the sample one wishes to estimate the parameters $\alpha_1, \alpha_2, \cdots, \alpha_p$ of the autoregressive process, $u_t+\alpha_1u_{t-1}+\cdots+\alpha_pu_{t-p}=v_t=$ pure white noise, and the standard deviations σ_u and σ_v. The estimation equations are usually quite cumbersome and the author discusses alternative methods of estimation. These methods are compared in terms of asymptotic efficiency. The effect of non-normality of the observations is discussed.

U. Grenander (Stockholm)

5109:

Roy, A. D. A note on prediction from an autoregressive process using pistimetric probability. J. Roy. Statist. Soc. Ser. B **22** (1960), 97–103.

The author observes that when jointly sufficient estimators of parameters exist for all sizes of samples, use can sometimes be made of Fisher's fiducial inference, in order to obtain the fiducial distribution of the parameters. In some cases, estimators which are jointly sufficient exist only for one size of sample. When this is the case the author suggests, given this size of sample, that the distribution of the parameters be obtained by a method analogous to fiducial inference, and then for larger samples by Bayes' theorem with the fiducial type probabilities as prior probabilities. He illustrates this procedure for the process $x_t=\beta x_{t-1}+\varepsilon_r$, where ε_r is normally and independently distributed over time with mean zero and variance σ^2. When, for example, $x_0=0$ at $t=1$ the maximum likelihood estimates for β and σ^2 based on two samples are sufficient, but not when based on larger samples.

J. L. Snell (Hanover, N.H.)

NUMERICAL METHODS

See also A4841, 5147, 5148, 5269, 5287, 5354, 5460, 5461, 5462.

5110:

Fisher, I. Z. Applications of the Monte Carlo method in statistical physics. Uspehi Fiz. Nauk **69** (1959), 349–369 (Russian); translated as Soviet Physics. Uspekhi **2** (1960), 783–796.

A survey article concerned with problems in statistical physics which have been treated by the Monte Carlo method. The conclusion of the author is that the Monte Carlo method is the method of greatest promise for a wide class of problems in statistical physics—a conclusion which the limited experience of this reviewer tends to support. *R. R. Coveyou* (Oak Ridge, Tenn.)

5111:

Golenko, D. I. Calculation of the characteristics of certain stochastic processes by the Monte Carlo method. Vyčisl. Mat. **5** (1959), 93–108.

A study of certain branching processes was carried out on the "Arrow" calculator at the USSR Academy of Sciences, and the present paper is a summary of the procedures used. The processes have a certain similarity to cosmic ray and gamma ray showers in which particles may split, change energy, die out, etc., on collision and disintegration. The author shows how the process can be synthesized by a Monte-Carlo (sampling) scheme, and the relevant random variables, integer-valued and exponentially distributed, can be generated, essentially by generation of "pseudo-random" numbers. Finally the cascade method, consisting of forty operations, is detailed step by step in arriving at the Monte-Carlo history of the process. *D. A. Darling* (Ann Arbor, Mich.)

5112:

Vionnet, Monique. Approximation de Tchebycheff d'ordre n des fonctions Arc sin x et Log x. Chiffres **3** (1960), 65–78. (English, German and Russian summaries)

The partial sums of Tchebycheff's expansions of two functions $\operatorname{Arcsin}(x/\sqrt{2})$ and $\operatorname{Log}(1+0.16x)$, for $0<x<1$ and $-1\leqq x\leqq 1$ respectively, are excellent approximations. They yield polynomial approximations in x, when Tchebycheff polynomials are replaced by their explicit expressions in terms of powers of x. The computation of resulting coefficients performed by the author has therefore a real practical value. Her tables are very accurate and they contain high order approximations (up to $n=51$ for the Arcsine and $n=20$ for the logarithm) which guarantee any number of correct digits after the dot not exceeding 23 for the Arcsine and 24 for the logarithm.

E. Kogbetliantz (New York)

5113:

Perlin, I. E.; Garrett, J. R. High precision calculation of Arcsin x, Arccos x, **and** Arctan x. **Math. Comput. 14** (1960), 270–274.

The polynomial approximation to Arctan x in the interval $(0, \tan \pi/24)$ studied in this paper is deduced from the expansion of Arctangent into a series of Chebyshev polynomials. The nine coefficients of the odd polynomial of seventeenth degree are given with 21 digits after the dot. Thus, the approximation is accurate to twenty decimal places in fixed point and to nineteen places in floating point arithmetics. For x in the infinite range $(\tan \pi/24, \infty)$ addition formulae are used in each of six subintervals as usual. The upper bounds of errors are well analyzed. Since only twenty stored constants are needed, the subroutines are very economical for the high accuracy achieved. The computation of Arcsine is reduced to that of the Arctangent and this requires additional multiplication, division and extraction of a square root.

E. Kogbetliantz (New York)

5114:

Davison, B. ★Some integrals involving the Bessel function integrals. CRT-856, AECL No. 867. Atomic Energy of Canada Limited, Chalk River, Ontario, 1959. 19 pp.

In this paper the author gives a systematic way of evaluating integrals of the type

$$F_{n,k}(x) = \frac{2}{\pi}\int_0^{\pi/2}\int_1^{\infty} \frac{\cos^k \phi}{t^n(t^2-1)^{1/2}} \exp\left[-2xt\cos\phi\right] dt d\phi,$$

where n and k are non-negative integers and $n+k$ is even. These integrals occur in the evaluation of neutron transport in cylindrical systems. They can be considered as generalized Bessel functions.

The function $F_{n,0}(x)$ satisfies a fourth order differential equation and it can be expressed as the sum of a polynomial and a sequence of Bessel functions. Recurrence relations for the coefficients in this sequence are given and the coefficients for the first eight terms of the expansion are tabulated for $n=0(2)8$.

L. J. Slater (Cambridge, England)

5115:

Morrison, David D. Remarks on the unitary triangularization of a nonsymmetric matrix. J. Assoc. Comput. Mach. **7** (1960), 185–186.

A method of triangularizing a matrix by applying a sequence of unitary multipliers was described by the reviewer [same J. **5** (1958), 339–342; MR **22** #1992] with the choice of certain quantities left optional. The author selects these for optimal stability of the computation. At any stage the method requires the selection of a vector u with $u^*u=2$, and a scalar ζ with $|\zeta|=1$, such that for a given vector a and assigned unit vector v, $(I-uu^*)a=\zeta\|a\|v$, the norm being Euclidean. The choice is

$$\zeta = -v^*a/|a^*v|, \quad u = (a-\zeta\|a\|v)/\mu,$$

where $\mu=[\|a\|(\|a\|+|a^*v|)]^{1/2}$.

A. S. Householder (Oak Ridge, Tenn.)

5116:

Akaike, Hirotugu; Saigusa, Yaeko. On a min-max theorem and some of its applications. Ann. Inst. Statist. Math. Tokyo **12** (1960), 1–5.

In the matrix A of order n, let the elements of each row satisfy $a_{ij}\geqq a_{i,j+1}$. It is required to permute the rows so that in the resulting matrix the largest diagonal element has its least possible value. The solution is obtained by placing first the row whose first element is smallest; placing second the row among those remaining whose second element is smallest; etc. At the jth step, if there are

equal jth elements the next ones are examined in an obvious way.

Two applications are made, one to an aspect of economic behavior: that of purchasing each year one of n desired objects. The other is to the use of the Gauss-Seidel method for solving $(I-D)x=b$. In terms of a suitable norm the convergence is optimal when the matrix whose elements are $a_{ij}=\sum_{k=j}^{n}|d_{ik}|/[1-\sum_{k=1}^{j-1}|d_{ik}|]$ is so ordered. The authors do not say so, but the columns of D must be permuted along with the rows. The first step is therefore to place first the row for which $\sum_{k=1}^{n}|d_{ik}|$ is a minimum (presumably $d_{ii}=0$) and to make a corresponding permutation of columns. *A. S. Householder* (Oak Ridge, Tenn.)

5117:

Karamyškin, V. V. On the choice of Galerkin functions in eigenvalue problems for finite-difference equations. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1958**, no. 6, 3–6. (Russian)

The author discusses a technique for approximating the eigenvalues for a finite difference equation of the form $[L-\lambda N]n=0$ (both L and N are finite difference operators), and applies it to an example.

R. R. D. Kemp (Kingston, Ont.)

5118:

Mayanc, L. S. Numerical solution and numerical analysis of the solutions to homogeneous systems of linear algebraic equations of general type. Dokl. Akad. Nauk SSSR **131** (1960), 51–54 (Russian); translated as Soviet Physics. Dokl. **5**, 257–260.

For the eigenvalue problem $(A-\lambda B)X=0$, if either A or B is nonsingular, an inversion reduces the problem to the standard one. Otherwise the problem is more complicated. The author observes that for nonsingular P and R the problem is equivalent to $(PAR-\lambda PBR)R^{-1}X=0$, hence selects P and R as unit lower triangular. Special cases arise, depending upon whether or not $AX=BX=0$ or $A^TY=B^TY=0$ or both can be satisfied nontrivially.

A. S. Householder (Oak Ridge, Tenn.)

5119:

Rutishauser, Heinz. Über eine kubisch konvergente Variante der LR-Transformation. Z. Angew. Math. Mech. **40** (1960), 49–54.

Convergence of the L-R-transformation can be improved by a suitable shift of the origin. However, for positive-definite matrices, this would require knowledge of a lower bound for the smallest eigenvalue in order to execute a Choleski decomposition. A surprisingly effective solution of the dilemma is given by the authors. Theorem 1: Decompose the hermitian matrix B, $B=\begin{pmatrix}U & V\\ V^H & W\end{pmatrix}$ and let $W^*=W-V^HU^{-1}V$. If $\lambda_{\min}(W^*)<0$, then $\lambda_{\min}(W^*)\leq\lambda_{\min}(B)$. If after a shift of the origin the Choleski decomposition breaks down, W^* is known to be indefinite, and determination of its smallest eigenvalue gives the next shift which makes a complete decomposition possible.

If the iteration has been carried through far enough, a shift which makes the last diagonal element equal to zero will cause a breakdown only in the last step, thus giving the bound immediately. It is shown that then the iteration process is cubically convergent. *F. L. Bauer* (Mainz)

5120:

Ehrmann, Hans. Konstruktion und Durchführung von Iterationsverfahren höherer Ordnung. Arch. Rational Mech. Anal. **4**, 65–88 (1959).

Der Verfasser behandelt Iterationsverfahren $x_{n+1}=f(x_n)$ höherer Ordnung zur Lösung einer Gleichung $F(x)=0$ mit einer Zahl x als Unbekannten. Zunächst werden Ergebnisse von E. Schröder [Math. Ann. **2** (1870), 317–365] zusammengestellt, um darauf hinzuweisen, dass es seit langem eine abgeschlossene Theorie der Aufstellung von Iterationsverfahren höherer Ordnung gibt und es theoretisch keine Schwierigkeiten macht, neue solche Verfahren anzugeben. Der Verfasser entwickelt dann Verfahren der Ordnung k ($k=2, 3, \cdots$), welche gegenüber den bekannten Methoden wesentliche praktische Vorteile haben: Die Rechnung ist erheblich einfacher, und man kann die Rechenarbeit unmittelbar abschätzen. Insbesondere lässt sich unter bestimmten Voraussetzungen eine Aussage über das günstigste Verfahren machen. Unter dem günstigsten Verfahren wird dabei dasjenige Verfahren r-ter Ordnung (der hier entwickelten Art) verstanden, bei dem man zur Erzielung einer Gesamtordnung $k=n^r$ in n Schritten mit der geringsten Zahl von Multiplikationen und Divisionen auskommt. Zur Lösung einer algebraischen Gleichung (mit $F'(\xi)\neq 0$ für die Lösung ξ) ist in diesem Sinne z.B. bis $n=5$ das Newtonsche Verfahren und für $n>5$ das Verfahren dritter Ordnung am günstigsten. Der Verfasser leitet ferner Fehlerabschätzungen her, welche die Ordnung des Verfahrens berücksichtigen und rechnet schliesslich zwei numerische Beispiele nach Verfahren verschiedener Ordnung.

J. Schröder (Hamburg)

5121:

Ghircoiaşiu, N. Une méthode de calcul approché des moments géométriques. Inst. Politehn. Cluj. Lucrări Şti. **1** (1958), 47–55. (Romanian. Russian and French summaries)

Elementary method for the computation of moments of an area, by dividing the area into narrow, parallel strips. The arithmetic is simplified by combinatorial considerations. *E. Grosswald* (Philadelphia, Pa.)

5122:

Flinn, E. A. A modification of Filon's method of numerical integration. J. Assoc. Comput. Mach. **7** (1960), 181–184.

This paper gives an approximation to an integral I of the following form:

$$\begin{aligned} I &= \int_A^B f(x)\cos px\,dx \\ &\doteq h\{S[f(B)\sin pB-f(A)\sin pA] \\ &\quad +hP[f'(B)\cos pB-f'(A)\cos pA] \\ &\quad +RC_{ce}+hQC_{se}'+NC_{co}+hMC_{so}'\}, \end{aligned}$$

where C_{co}, C_{so}', C_{ce}, and C_{se}' are sums of certain values of $f(x)\cos px$, $f(x)\sin px$, $f'(x)\cos px$, $f'(x)\sin px$ and the other unidentified quantities are specified rational functions of $\theta=hp$. These results are stated to be special instances of those due to Luke [Proc. Cambridge Philos. Soc. **50** (1954), 269–277; MR **15**, 992].

P. C. Hammer (Madison, Wis.)

5123:

Gorbunov, A. D.; Budak, B. M. On the stability of numerical processes arising in the solution by many-point difference methods of the Cauchy problem for the equation $dy/dx=f(x, y)$. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 2, 15–23. (Russian)

Let $y(x)$ be the solution of the problem $dy/dx=f(x, y)$, $y(x_0)=y_0$, where $f(x, y)$ is Lipschitzian and computable to any desired number of decimal places in some rectangle $|x-x_0|\leqq A$, $|y-y_0|\leqq B$. Let the values of $y(x_0+kh)$ $(k=1, 2, \cdots)$ be approximated by the quantities $y_1, y_2, \cdots$, computed from

$$\sum_{i=0}^{m} \alpha_i y_{k-i} = h \sum_{i=0}^{n} \beta_i f(x_{k+l-i}, y_{k+l-i}), \quad \alpha_0, \beta_0, \alpha_m, \beta_n \neq 0,$$

$(x_j=x_0+jh)$ with appropriately chosen initial values for $y_0, y_{-1}, \cdots$. It is supposed that a number $\bar{x}$, $x_0<\bar{x}\leqq x_0+A$ exists such that, for all sufficiently small h, y_k can be computed for $k=1, 2, \cdots, S_h$, where $S_h=1+(\bar{x}-x_0)/h$. The finite-difference scheme described above is carried out numerically with rounding-off, the rounded-off results being denoted by $y_1^*, y_2^*, \cdots$. For each k let $D_k=y(x_k)-y_k^*$, $d_k=y_k-y_k^*$, and $\delta_k=y(x_k)-y_k$. The process is said to converge in case $D_k\to 0$ as $h\to 0$ uniformly in k on $0\leqq k\leqq S_h$ in a certain sense. The process is said to be stable of order ν $(\nu=0, 1)$ if $\Delta^\nu d_{k-1}=o(h^\nu)$ as $h\to 0$ uniformly in k for $0\leqq k\leqq S_h+(1-\nu)$.

The authors give a number of results about the stability of convergent processes. The precise statements are too complicated to give here.

W. S. Loud (Minneapolis, Minn.)

5124:

Wilf, Herbert S. Maximally stable numerical integration. J. Soc. Indust. Appl. Math. 8 (1960), 537–540.

This paper examines the question of the maximum radius of stability for corrector formulas for integrating first order differential equations using 2- and 3-point formulas. The paper raises four questions of importance for further research. *R. W. Hamming* (Stanford, Calif.)

5125:

Kadner, Horst. Untersuchungen zur Kollokationsmethode. Z. Angew. Math. Mech. **40** (1960), 99–113.

Let $[a, b]$ be a finite interval, let the functions f_ν $(\nu=0, 1, \cdots, n)$ and f_n^{-1} be in $C^n[a, b]$, and let r be in $C[a, b]$. The author finds an approximate solution of the boundary value problem

$$L(y) = \sum_{\nu=0}^{n} f_\nu(x)y^{(\nu)}(x) = r(x), \qquad x\in[a, b],$$

$$U_\mu(y) = \sum_{k=0}^{n-1} [\alpha_{\mu k}y^{(k)}(a)+\beta_{\mu k}y^{(k)}(b)] = 0, \quad \mu = 1, 2, \cdots, n,$$

by picking k coordinate functions $r_j(x)$ satisfying the boundary conditions $U_\mu(r_j)=0$ $(\mu=1, 2, \cdots, n;\ j=1, 2, \cdots, k)$ and determining constants c_j such that

$$L\Big(\sum_{j=1}^{k} c_j r_j\Big) = r(x_i)$$

at p distinct points $x_i\in[a, b]$. He shows that this scheme can be thought to arise from minimizing

$$\int_a^b \Big[L\Big(\sum_{j=1}^{k} c_j r_j\Big)-r\Big]^2\, dx$$

if the integral is evaluated by a quadrature formula. A bound for the mean square error, minimized for $p=k$ fixed if the x_i are selected as abscissas of a Gaussian quadrature formula, is derived. Two numerical examples are given. *P. Henrici* (Los Angeles, Calif.)

5126:

Fox, Leslie. Some numerical experiments with eigenvalue problems in ordinary differential equations. Boundary problems in differential equations, pp. 243–255. Univ. of Wisconsin Press, Madison, 1960.

The author describes numerical experiments on methods for solving eigenvalue problems involving ordinary differential equations on a high-speed computer. The basic idea is one of "shooting". The differential equation is solved as an initial value problem for guessed values of the initial data and of the eigenvalue, and subsequent appropriate changes in the guessed values are determined so that boundary conditions or continuity conditions are ultimately satisfied. The changes are determined by Newton's method, the necessary derivatives being calculated as solutions of certain variational equations. The paper discusses techniques for solving a single second-order equation, a single fourth-order equation, and up to three simultaneous second-order equations. The latter examples have a boundary condition at infinity, and a method is shown to overcome certain difficulties in the determination of the eigenfunctions that are likely to arise in this case. The result of the thoroughly practical and workmanlike discussion is that the shooting technique is satisfactory in the examples considered, the convergence being ultimately quadratic in each case.

P. Henrici (Los Angeles, Calif.)

5127:

Young, David; Ehrlich, Louis. Some numerical studies of iterative methods for solving elliptic difference equations. Boundary problems in differential equations, pp. 143–162. Univ. of Wisconsin Press, Madison, 1960.

Solving difference equations arising from elliptic partial differential equations in two space variables, such as the Dirichlet problem, by iterative methods can be very time consuming even on modern digital computers if the mesh spacings of the discrete approximation are small. Two iterative methods currently in wide use are the successive overrelaxation method of Young and Frankel, and the Peaceman-Rachford method, a particular variant of implicit alternating direction methods. As the theory for the rigorous applications of these methods unfortunately as yet does not allow us to theoretically compare these methods except for very special problems (i.e., essentially the Dirichlet problem for a rectangle with uniform mesh spacings), the authors compare these methods for a series of numerical problems in an attempt to shed light, for example, on the extent to which numerical results derived from more general problems agree with theoretical results derived under special assumptions. Their results seem to indicate numerically that the known theoretical results for the Peaceman-Rachford method are applicable to a much wider class of problems, although it is known [G. Birkhoff and R. S. Varga, Trans. Amer. Math. Soc. **92** (1959), 13–24; MR **21** #4549] that the method of proof of theoretical results for this method still restricts one to rectangular mesh regions. Moreover, for large numbers of

mesh points, the numerical results indicate a large advantage in favor of the Peaceman-Rachford iterative method over the point successive overrelaxation iterative method.

The reviewer, along with J. Douglas, Jr., feels that the error criterion used in these numerical experiments to terminate iterations was poorly chosen and might affect the comparison. Moreover, the reviewer wonders why some newer, more competitive, variant of successive overrelaxation, such as line or block successive overrelaxation, also widely used, was not also considered.

R. S. Varga (Cleveland, Ohio)

5128:

Práger, Milan. Sur une modification de la méthode de M. Kantorovitch. Apl. Mat. **5** (1960), 305–316. (Czech and Russian summaries)

A method of Kantorovič for approximate solution of the first boundary value problem for the elliptic equation

$$\frac{\partial}{\partial x}\left(a(x, y)\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(b(x, y)\frac{\partial u}{\partial y}\right)-c(x, y)u = f(x, y),$$

as presented in the book of Kantorovič and Krylov, *Približennye metody vysšego analiza* [Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1950; MR **13**, 77], is modified so that higher approximations may be obtained by solving each time a single ordinary differential equation rather than a system of such equations as required in the original method. Conditions for the validity of the modified method are stated and a proof of convergence of the modified Kantorovič sequence is given. Two numerical examples, one for Poisson's equation and the other for an extension of the method to the biharmonic equation, are given.

J. F. Heyda (Cincinnati, Ohio)

5129:

Milnes, Harold W.; Potts, Renfrey B. Numerical solution of partial differential equations by boundary contraction. Quart. Appl. Math. **18** (1960/61), 1–13.

A new method is proposed for the numerical solution of linear boundary value problems in two-dimensional regions bounded by a Jordan curve S_0. It is based on difference approximations in a grid whose nodes are the intersections of a family of nested contours $S_0, S_1, S_2, \cdots$ and "radial" curves so as to obtain a pattern of curves topologically equivalent to the coordinate grid of polar coordinates. The difference equations considered permit the successive calculation of their solution on the level curves $S_1, S_2, \cdots$, by solving each time a system of linear algebraic equations. For problems with constant coefficients the matrices involved in these calculations are circulants, and an explicit analysis of the stability conditions is possible. The paper deals almost exclusively with difference equations as such. One differential equation problem is studied as an example. It is of hyperbolic type. According to the authors the stability conditions are not easy to fulfill for elliptic problems.

W. Wasow (Madison, Wis.)

5130:

Newman, D. J. Numerical method for solution of an elliptic Cauchy problem. J. Math. and Phys. **39** (1960/61), 72–75.

The elliptic Cauchy problem of the title is $\partial^2\phi/\partial x^2+\partial^2\phi/\partial y^2=0$, $-\infty<x<\infty$, $0<y\leq 1$, $\phi(x, 0)=F(x)$, $\phi_y(x, 0)=G(x)$. It is supposed to have a solution possessing a certain number of bounded derivatives. The basic idea of the numerical method is to combine a difference procedure in the y-direction with a smoothing at each step by means of the integral transformation defined by

$$f_T(x) = \frac{2}{\pi T}\int_{-\infty}^{\infty} f(t+x)(\sin^2 Tt-\sin^2 \tfrac{1}{2}Tt)t^{-2}\,dt.$$

Functions $u(x, y)$, $v(x, y)$ are then recursively defined for $y=0, \Delta, 2\Delta, \cdots$ by

$$u(x, 0) = F_T'(x),\quad v(x, 0) = G_T(x),$$
$$u(x, y_0+\Delta) = u(x, y_0)+\Delta(d/dx)v_{2T}(x, y_0),$$
$$v(x, y_0+\Delta) = v(x, y_0)-\Delta(d/dx)u_{2T}(x, y_0).$$

The author proves that u and v are, for small Δ and large T, approximations to ϕ_x and ϕ_y, respectively, and that the method is stable with respect to round-off errors.

W. Wasow (Madison, Wis.)

5131:

Volkov, E. A. Investigation of a method for increasing the accuracy of the method of nets in the solution of the Poisson equation. Vyčisl. Mat. **1** (1957), 62–80. (Russian)

Investigation of a method of improving the finite-difference solution of the Dirichlet problem for the Poisson equation $\Delta u=\varphi$, $u|_\Gamma=f$ in a domain G with curvilinear boundary Γ; the gist is as follows.

Take the difference operator $D^{(n)}$ which represents the difference between the Laplace differential operator Δ and the elementary difference operator Δ_h on a square net accurately to quantities of order h^{2n} (h is the mesh of the net); then solve by the method of successive approximations the difference equation

$$\text{(1)}\qquad \begin{aligned}\Delta_h u_{ij}^{(k+1)} &= h^2\varphi_{ij}+D^{(n)}u_{ij}^{(k)},\\ \Delta^{-(2n-2)}u_{ij}^{(k+1)} &= 0,\quad u^{(k+1)}|_\Gamma = f.\end{aligned}$$

Here $\Delta^{-(2n-2)}$ is the operator of deviation of the boundary function at the corners of the grid contour, representing a certain interpolation formula based on Lagrange's, and having remainder term of order h^{2n-2}. This interpolation formula can have, in general, remainder term of arbitrarily high order in h, and satisfies the sufficient conditions for applicability of the method of iteration for the solution of the difference equation.

Two lemmas are established: on uniqueness of the solution of the difference equation, and on an estimate of the error in the solution arising from replacing Δ by a difference operator.

Two theorems are proved concerning the method. Theorem 1 states that the solution $u^{(1)}$ of

$$\text{(2)}\qquad \Delta_h u_{ij}^{(1)} = h^2\varphi_{ij},\quad \Delta^{-(2n-2)}u_{ij}^{(1)} = 0,\quad u^{(1)}|_\Gamma = f$$

has the form

$$u_{ij}^{(1)} = u_{ij}+\sum_{s=1}^{n-2} h^{2s}(W_s^{(1)})_{ij}+O(h^{2n-2}),$$

where u is the exact solution of the problem and $W_k^{(1)}$ is some function independent of h and sufficiently smooth in G. Here it is required that the boundary Γ and the functions φ and f be sufficiently smooth.

Theorem 2 asserts that, under the conditions of theorem

1, given the kth approximant $u^{(k)}$ to the exact solution u, related to u by

$$(3) \qquad u_{ij}^{(k)} = u_{ij} + \sum_{s=k}^{n-2} h^{2s}(W_s^{(k)})_{ij} + O(h^{2n-2k}),$$

the solution $u^{(k+1)}$ of equation (1) is the $(k+1)$st approximant to u related to it by (3) with k replaced by $k+1$. $W_s^{(k)}$ is some function independent of h and sufficiently smooth in G.

On the basis of theorem 2 it is remarked that (a) the kth approximant $u^{(k)}$ (for $k \le [\frac{1}{2}n]$, n fixed) converges to the exact solution u as h^{2k}, and for sufficiently small h approximates it better (uniformly over the nodes) than the $(k-1)$st; and (b) $u^{(k)}$ may be numerically differentiated, the error in computation of the rth derivative of u being of order at most $\max\{h^{2k}, h^{2n-2k-r}\}$.

It is remarked that in case Γ coincides with the grid contour equation (2) simplifies, in that the operator $\Delta^{-(2n-2)}$ vanishes, but theorems 1 and 2 are not proved for such regions.

In conclusion some numerical examples are given.

D. F. Davidenko (RŽMat **1958** #9248)

5132:

Wachspress, E. L.; Habetler, G. J. An alternating-direction-implicit iteration technique. J. Soc. Indust. Appl. Math. **8** (1960), 403–424.

The authors extend the theory and efficient numerical application of the Peaceman-Rachford iterative method, a particular variant of the implicit alternating direction methods, in several ways. If the matrix problem to be solved is (1) $(H+V+B)\mathbf{x}=\mathbf{k}$, then the Peaceman-Rachford iterative method defined by

$$(2) \qquad \begin{aligned} (H+B+\omega_n I)\mathbf{x}^{(n+1/2)} &= (\omega_n I - V)\mathbf{x}^{(n)} + \mathbf{k}, \\ (V+B+\omega_n I)\mathbf{x}^{(n+1)} &= (\omega_n I - H)\mathbf{x}^{(n+1/2)} + \mathbf{k} \end{aligned}$$

is shown to be convergent for $\omega_n \equiv \omega$ under more general conditions than given previously [see Birkhoff and Varga, Trans. Amer. Math. Soc. **92** (1959), 13–24; MR **21** #4549]. Next, assuming that H, V, and B are symmetric and positive definite matrices with (3) $HV=VH$, $HB=BH$, $VB=BV$, the so-called "model problem", the authors use the Chebyshev principle applied to rational functions to determine theoretically the best parameters ω_n to be used cyclically in (2). But their contribution contains several practical points. First, they condition their matrices by the construction of positive diagonal matrices F, so that the property of (3), with $\bar{H}=FHF$ replacing H, etc., is more "nearly" satisfied. This construction is given for discrete approximations to the general class of problems (4) $-\operatorname{div}\{D \operatorname{grad} \phi\}+\sigma\phi=S$. Also, they indicate how storage requirements and arithmetic can be efficiently reduced in actual applications of the Peaceman-Rachford method. Finally, in an Appendix A, they prove that the conditions of (3) with $\bar{H}$ replacing H, etc., are valid for the case of the Dirichlet problem even with non-uniform mesh spacings in the rectangle. Non-rectangular regions still pose a problem.

R. S. Varga (Cleveland, Ohio)

5133:

Schechter, Samuel. Quasi-tridiagonal matrices and type-insensitive difference equations. Quart. Appl. Math. **18** (1960/61), 285–295.

The author extends the work of Karlqvist [Tellus **4** (1952), 374–384; MR **15**, 166] and Cornock [Proc. Cambridge Philos. Soc. **50** (1954), 524–535; MR **16**, 180] in directly inverting quasi-tridiagonal matrices of the form

$$Q = \begin{bmatrix} M_1 & E_1 & & & \mathbf{0} \\ D_2 & M_2 & E_2 & & \\ & \cdot & \cdot & \cdot & \\ & & \cdot & \cdot & E_{t-1} \\ \mathbf{0} & & & D_t & M_t \end{bmatrix}.$$

In so doing, he gives a criterion for doing this which is similar to the LDU theorem. While Karlqvist and Cornock gave applications to the direct solution of Poisson and biharmonic problems, the author applies this method in particular to the Tricomi equation. Numerical results with this method are included for the Tricomi equation, and indicate good results for problems with not too large a number of mesh points.

R. S. Varga (Cleveland, Ohio)

5134:

Douglas, Jim, Jr. A numerical method for the solution of a parabolic system. Numer. Math. **2** (1960), 91–98.

The author introduces a finite difference method for the approximate solution of a parabolic system of differential equations arising from two-phase flow of incompressible fluids in a multi-dimensional porous medium. The mathematical model for this physical problem is given by the non-linear differential system

$$(1) \qquad \begin{aligned} \nabla\cdot\{\alpha\nabla u\}+\nabla\cdot\{\beta\nabla v\} &= 0, \\ \nabla\cdot\{\beta\nabla u\}+\nabla\cdot\{\alpha\nabla v\} &= \gamma\partial v/\partial t, \end{aligned}$$

where the coefficients α, β, and γ are given functions of position and the dependent variable $v(x, y, t)$. By considering instead the simpler equations

$$(2) \qquad \begin{aligned} \alpha\nabla^2 u+\beta\nabla^2 v &= 0, \\ \beta\nabla^2 u+\alpha\nabla^2 v &= \gamma\partial v/\partial t, \end{aligned}$$

which can be algebraically reduced to the form

$$(3) \qquad \begin{aligned} (1-a^2)\nabla^2 v &= b\partial v/\partial t, \\ \nabla^2 u+a\nabla^2 v &= 0, \end{aligned}$$

the author considers using the implicit backward-time method for stepping ahead in time the numerical approximation of $v(x, y, t)$, and the alternating direction method of Peaceman and Rachford for finding the numerical approximations of $u(x, y, t)$ from the second equation of (3), i.e.,

$$(3') \qquad \begin{aligned} (1-a^2)A\mathbf{V}^{(n+1)} &= b(\mathbf{V}^{(n+1)}-\mathbf{V}^{(n)})/\Delta t \\ A\mathbf{U}^{(n+1)}+aA\mathbf{V}^{(n+1)} &= \mathbf{0}, \end{aligned}$$

where A is the discrete matrix approximation to the Laplace operator ∇^2. Making strong assumptions, such as that u and v are four-times boundedly differentiable in essentially the region: $0 \le x,\ y \le 1,\ 0 \le t \le T$, the author proves that the numerical approximations U and V converge to u and v, i.e.,

$$(4) \qquad \begin{aligned} \max|v-V| &= O((\Delta x)^2+\Delta t) \quad (0 \le t \le T), \\ \max|u-U| &= O(\Delta x). \end{aligned}$$

Finally, estimates of the total labor involved in carrying out this two-stage numerical process are given, the author making assumptions necessary for the validity of the commutative theory of implicit alternating direction methods. [See G. Birkhoff and R. S. Varga, Trans. Amer. Math. Soc. **92** (1959), 13–24; MR **21** #4549.]

R. S. Varga (Cleveland, Ohio)

5135:

Muhina, G. B.; Sobolev, S. L. On the solution of a boundary value problem. Prikl. Mat. Meh. **23** (1959), 534–539 (Russian); translated as J. Appl. Math. Mech. **23**, 754–761.

Let (r, θ, z) denote cylindrical coordinates, and denote by D_1, D_2, D_3 the regions defined by $|z|<h$, $r<r_0$; $|z|<H$, $r_0<r<R$; $h<|z|<H$, $r<r_0$, respectively, where $0<h<H$, $0<r_0<R$. The authors consider the problem (apparently inspired by reactor theory) of finding the smallest real number ν admitting the existence of non-zero functions ϕ_i and m_i $(i=1, 2, 3)$ satisfying the differential equations

$$\begin{aligned}\Delta\phi_i+a_{11i}\phi_i+a_{12i}m_i &= 0,\\ \Delta m_i+\delta_{1i}\nu\phi_1+a_{22i}m_i &= 0,\end{aligned}\qquad (r, \theta, z)\in D_i,$$

(a_{jki}, b_{jki} = given constants, δ_{jk} = Kronecker delta) and certain simultaneous homogeneous conditions too complicated to be stated here at the interfaces and boundaries. Using Hilbert-Schmidt integral equation theory, they construct a complete system of solutions of the differential equations (expressed in terms of Bessel and trigonometric functions) satisfying all boundary conditions except those at $r=r_0$. The eigenvalue ν is then determined by the requirement that a linear combination of these particular solutions must satisfy also this last condition. Numerical computations using six to nine coordinate functions in each of the subregions D_i are said to have been carried out on the BESM computer, but details are not given.

P. Henrici (Los Angeles, Calif.)

5136:

Džems-Levi, G. E. On functions whose nomograms have a given answer scale. Vyčisl. Mat. **5** (1959), 109–132.

The author starts out by giving the conditions necessary for the existence of a nomographic factor $A(x, y)$ in order that the left-hand side of the relation $f_3(z)K(x, y)+\varphi_3(z)L(x, y)+\psi_3(z)M(x, y)=0$ be identically equal to

$$\begin{vmatrix} f_1(x) & \varphi_1(x) & 1\\ f_2(y) & \varphi_2(y) & 1\\ f_3(z) & \varphi_3(z) & 1\end{vmatrix}.$$

As examples, it is shown how to bring the equations of the 5th and 4th nomographic order, respectively,

$$R(x) = \frac{S(z)-T(y)}{U(z)-V(y)} \text{ and } f_3L_{12}+\varphi_3M_{12}+\psi_3N_{12} = 0,$$

where

$$\begin{aligned}L_{12} &= l_0f_1f_2+l_1f_1+l_2f_2+l_3,\\ M_{12} &= m_0f_1f_2+m_1f_1+m_2f_2+m_3,\\ N_{12} &= n_0f_1f_2+n_1f_1+n_2f_2+n_3,\end{aligned}$$

to the form of the above determinant. He then treats nomograms with a given rectilinear answer scale. The method is illustrated on the equation $x+y=z$. For the case when the given equation cannot be nomographed with a uniform rectilinear answer scale, the author gives an approximate method, and discusses the exactness of the method.

D. Mazkewitsch (Cincinnati, Ohio)

5137:

Reutter, F. Theorie der Fluchtliniennomogramme für Systeme von zwei Funktionen zweier reeller Veränderlichen. Z. Angew. Math. Mech. **40** (1960), 75–93.

From the author's summary: "Starting with the representation of a system of two functions of two real variables by means of two families of glide curves, the necessary and sufficient conditions are given for two functions $x=x(u, v)$, $y=y(u, v)$ to be representable by a (common) alignment chart; a general method is developed for determining the scale equations of such a nomogram. From this theory a different solution for the basic problem, treated systematically by T. H. Gronwall, of nomographic representation of one function of two variables is obtained; results are reached which I. A. Vilner published in various Russian magazines.

"A detailed investigation is made of systems of such functions which are connected by a special system of two linear partial differential equations of 1st order with constant coefficients. As application, all the representable functions $w=f(z)$ of a complex variable are tabulated in two tables."

D. Mazkewitsch (Cincinnati, Ohio)

5138:

Džems-Levi, G. E. Some general methods in practical nomography. Vyčisl. Mat. **4** (1959), 104–149. (Russian)

The author considers only alignment charts. He solves in principle the problem of anamorphosis. The paper consists of three chapters. Chapter I deals with the problem of general anamorphosis: construction of a nomogram by intersecting points; differential equations of the scales of a nomogram; nomograms for systems of equations; application of results obtained to equations of 3rd nomographic order; equations of 5th nomographic order; nomograms containing a rectilinear scale for systems of equations; nomograms with a rectilinear answer scale for the equation of 3rd nomographic order (it is shown that it is enough to consider only three answer scales: $f_1(x)=e^{ax}$, $f_1(x)=x$, $f_1(x)=\tan ax$ (a a parameter); other scales are obtained by projective transformation); condition for reducing $z=F(x, y)$ to a nomogram of 3rd nomographic order. Chapter II deals with nomographing without quadrature. Considered are: equations of 5th nomographic order, and a method of approximate nomographing. Chapter III deals with transformation of nomograms: projective transformations with a fixed triangle, and apparatus for transformation of nomograms of 3rd nomographic order.

This highly interesting paper is purely theoretical. The reader has to apply the theory presented to particular cases.

D. Mazkewitsch (Cincinnati, Ohio)

5139a:

Lebedev, A. V.; Fedorova, R. M. ★A guide to mathematical tables. English edition prepared from the Russian by D. G. Fry. Pergamon Press, New York-London-Oxford-Paris, 1960. xlvi+586 pp. $15.00.

5139b:

Burunova, N. M. ★A guide to mathematical tables: Supplement No. 1. Supplement to "A guide to mathematical tables" by A. V. Lebedev and R. M. Fedorova. English edition prepared from the Russian by D. G. Fry. Pergamon Press, New York-Oxford-London-Paris, 1960. xxxviii+190 pp. $9.00.

The original Russian edition [Izdat. Akad. Nauk SSSR, Moscow, 1956] of the main work was reviewed in MR **18**, 828; the original Russian edition of the supplement [Izdat. Akad. Nauk SSSR, Moscow, 1959] was reviewed in MR **22** #2030. The tabular matter has been reproduced directly from the original, but the titles in Cyrillic script in the reference section have been translated.

The supplement contains information on tables which have been published in the U.S.S.R. and abroad since the publication of the main work and also tables which did not find their way into it. The arrangement of material is the same as in the Guide.

5140:

Jahnke, Eugene; Emde, Fritz; Lösch, Friedrich. ★Tables of higher functions. 6th ed. Revised by Friedrich Lösch. McGraw-Hill Book Co., Inc., New York-Toronto-London; B. G. Teubner Verlagsgesellschaft, Stuttgart; 1960. xiv+318 pp. $14.00.

The third edition [Jahnke and Emde, *Tables of functions with formulae and curves*, Dover, New York, 1943], of this well-known work was reviewed in MR 4, 281 (errata in MR 4, 340) and the fourth edition [Dover, New York, 1945] in MR 7, 485. The present, sixth, edition is a completely revised and enlarged version, some of the enlargements are the following: there are new tables of the error-function and its derivative; the table of Fresnel integrals has been enlarged; there are improvements in the tables of elliptic functions, the theta function and Bessel functions. The section on confluent hypergeometric functions has been rewritten and the sections on Laguerre and Hermite polynomials are collected in a section on orthogonal polynomials and supplemented by sections on Tschebyscheff polynomials. Newly calculated tables of the Planck-Einstein and Debye functions are included.

COMPUTING MACHINES

See also 5111, 5113, 5196, 5214, 5503, 5504.

5141:

Buckingham, R. A. The organization of a university computing centre. Comput. J. **3** (1960/61), 131–135.

Author's summary: "In this paper, which was presented at the Harrogate Conference of The British Computer Society on 6 July 1960, the aims of a university computing centre are discussed, and a brief description given of the development and organization of the University of London Computer Unit."

5142:

Clarke, Laurence. Notes on the state of digital computing in the U.S.S.R. Comput. J. **3** (1960/61), 164–167.

Author's summary: "This paper, which was presented at the Harrogate Conference of The British Computer Society on 6 July 1960, gives an informal account of the state of computing in the U.S.S.R. as it appeared during a visit of 14 days to Moscow in June 1960."

5143:

Douglas, A. S.; Mitchell, A. J. Autostat: a language for statistical data processing. Comput. J. **3** (1960/61), 61–66.

A language for expressing the operation of statistical data processing (e.g., market research surveys) in a form which can be easily translated into computer code, is described. Pseudo-operations for inputting data from questionnaires, defining tables, grouping and weighting data, specifying the output format and performing standard computation in statistics (e.g., analysis of variance) are provided. A translator has been written for converting statements expressed in this language into the code of the Ferranti Pegasus computer.

C. C. Gotlieb (Toronto)

5144:

Duncan, F. G.; Huxtable, D. H. R. The DEUCE Alphacode translator. Comput. J. **3** (1960/61), 98–107.

Autocodes written for a single level store machine are inefficient when applied to machines having several storage levels; they are, however, attractive to the user because of their simplicity. The paper describes what the authors believe to be the first successful attempt to construct an efficient translator of this type.

The result, which contains 22,000 instructions and took about four man-years to write, is about 6 times as fast as a translator programme which did not attempt a re-interpretation of storage type.

The paper forms a very readable introduction to the principles involved in the construction of the new programme.

A. D. Booth (London)

5145:

Swift, J. D. Isomorph rejection in exhaustive search techniques. Proc. Sympos. Appl. Math., Vol. 10, pp. 195–200. American Mathematical Society, Providence, R.I., 1960.

Critique of methods used to reject isomorphic (partial or complete) solutions in machine search for solutions to various combinatorial problems. Examples include orthogonal latin squares, Veblen-Wedderburn systems, finite projective planes, finite semigroups, Steiner triple systems.

5146:

Gleason, Andrew M. A search problem in the n-cube. Proc. Sympos. Appl. Math., Vol. 10, pp. 175–178. American Mathematical Society, Providence, R.I., 1960.

Report on a computer experiment on the method of ascent applied to the following problem: Given a matrix consisting of ± 1's, maximize the sum of the entries by (repeated) application of the operation of multiplying a row or column by -1.

5147:

Fisher, Michael E. Limitations due to noise, stability

and component tolerance on the solution of partial differential equations by differential analysers. J. Electronics Control (1) 8 (1960), 113–126.

This paper is concerned with the analysis of errors in various techniques for the approximate solution of partial differential equations on electronic differential analysers. The techniques considered are the replacement of derivatives with respect to all but one of the variables by finite difference approximations; the approximations may be crude (first order), or higher order difference corrections may be taken into account. The methods then divide naturally into two categories: (i) the serial method of solving consecutively a sequence of ordinary differential equations in the remaining independent variable; (ii) the parallel method of solving simultaneously a set of N ordinary differential equations in the remaining independent variable. [Cf. D. R. Hartree, *Calculating instruments and machines*, Ch. 3, University Press, Cambridge, 1950; MR **12**, 133.]

Each of these methods is examined for parabolic, hyperbolic and elliptic partial differential equations in two variables. The accuracy and stability of the methods are considered and the effect of component variations is studied. It is concluded that stable parallel methods are available for parabolic and (with severe restrictions on attainable accuracy) hyperbolic partial differential equations; the techniques are not practicable for elliptic equations. *J. G. L. Michel* (Teddington)

5148:

Neustadt, Lucien. A method of computing eigenvectors and eigenvalues on an analog computer. Math. Tables Aids Comput. **13** (1959), 194–201.

For a real and symmetric $n \times n$ matrix A the author proposes to compute the eigenvector x of its largest eigenvalue (multiplicity one is assumed) by means of an electronic analogue computer solving the differential equations $dx/dt = 2Ax + kf(|x|)x$; $f(|x|) = \operatorname{sgn}(1-|x|^2)$. The constant k is positive and greater than twice the largest modulus of the eigenvalues. The method can be extended to compute the eigenvector of the smallest eigenvalue ($-A$ substituted for A). It is also possible to find the other eigenvectors by changing the differential equation so that it leads to solutions which are orthogonal to already known eigenvectors. Numerical experiments for 3×3 and 6×6 matrices show that a three-place accuracy can be obtained for the eigenvectors and a six-place accuracy for eigenvalues computed from the eigenvectors.

It is hard to see why the aforementioned differential equation for x should lead to an eigenvector, since $d|x|^2/dt < 0$ for $|x| > 1$ and > 0 for $|x| < 1$. Once the computer reaches a phase with $|x| = 1$ it must stay that way, and the k-term would not matter. But in practice the switching device representing $f(|x|)$ never works accurately, hence oscillations around the phase $|x| = 1$ take place, and it appears that these oscillations account for the success of the method. *H. Bückner* (Madison, Wis.)

MECHANICS OF PARTICLES AND SYSTEMS

See also 5456.

5149:

Symon, Keith R. ★Mechanics. 2nd ed. Addison-Wesley Series in Physics. Addison-Wesley Publishing Co., Inc., Reading, Mass.-London, 1960. xiv + 557 pp. $10.50.

The first seven chapters of this book treat the standard topics of the mechanics of particles and rigid bodies which usually make up a one-semester introductory course. Chapter 8 on continuous media, and Chapter 9 on Lagrange's equations, stopping short of variational methods, give the introduction desirable in such a course. The last three chapters, dealing with tensors, rotation of a rigid body, and small vibrations have been added in this second edition with the intent of providing material for a second semester's work.

From the beginning, some indications are given as to the modifications made in going from classical Newtonian to relativistic or to quantum mechanics. And many problems of interest in atomic physics are solved by using models based on Newtonian mechanics.

Mathematical topics beyond elementary calculus, such as vector analysis, differential equations, and Fourier analysis are explained as needed, with references to fuller treatments. The mathematics is usually adequate and as rigorous as it pretends to be. But the statement on Fourier analysis of merely continuous functions on pages 60, 299, 300, as well as the statement on page 428 that a non-symmetric third order tensor cannot have all three eigenvalues real, is incorrect.

The treatment is well suited for an average class. Although a little slow in pace for bright, well-prepared juniors, such students will find plenty of starred problems to challenge their best efforts. *P. Franklin* (Cambridge, Mass.)

5150:

Ziegler, Hans. ★Mechanik. Bd. I. Statik der starren und flüssigen Körper sowie Festigkeitslehre. 3te, neubearbeitete Aufl. Lehr- und Handbücher der Ingenieurwissenschaften, Bd. 5. Birkhäuser Verlag, Basel-Stuttgart, 1960. 244 pp. Fr./DM 28.50.

Der jetzt in dritter Auflage erschienene erste Band des bekannten Werkes enthält drei Kapitel: I. Statik der starren Körper, II. Statik der Flüssigkeiten, III. Festigkeitslehre. In ungewöhnlich klarer und didaktisch ausgefeilter Darstellung wird hier eine Einführung in die Grundlagen der Mechanik gegeben, die man jedem Studierenden empfehlen kann. Jede überflüssige Breite im Stofflichen wie in der Darstellung wird vermieden, um die wesentlichen Grundgedanken deutlich herauszuarbeiten. So sind gegenüber den ersten beiden Auflagen einige Methoden der graphischen Statik fortgelassen, die mehr aus historischem Interesse als aus zwingender Notwendigkeit in vielen Büchern ausführlich dargestellt sind. Eine deutliche Verschiebung von den graphischen zu den analytischen Methoden ist festzustellen. Abweichend von dem sonst vielfach gewählten Aufbau ist die Behandlung der Hydrostatik im Rahmen der allgemeinen Statik. Das könnte für den Anfänger vielleicht zu Schwierigkeiten führen, da bei der Behandlung der Statik von Kraftfeldern bereits Begriffe aus der Vektoranalysis verwendet werden. Auch findet man jetzt die Flächenträgheitsmomente in der Hydrostatik und nicht—wie sonst üblich—in der Elastostatik behandelt. Auch für die Darstellung der Festigkeitslehre ist die Konzentration auf grundlegende Tatsachen und Ansätze vor der Beschreibung von Sonderfällen bevorzugt. Spannungs- und Verzerrungs-

zustand werden für ein einführendes Werk erstaunlich umfassend behandelt. Man findet erfreulicherweise auch einen Überblick über die Ansätze der Plastizitätstheorien.

Neu gegenüber den früheren Auflagen sind Übungsaufgaben, für die freilich weder der Lösungsweg noch das Ergebnis angegeben wird. Die Ausstattung des Buches ist ausgezeichnet; vor allem zeichnen sich die Abbildungen durch große Anschaulichkeit aus. Man kann dieses Buch ohne Zweifel zu den besten einführenden Lehrbüchern der Mechanik zählen. *K. Magnus* (Stuttgart)

5151:

Thun, R. E. On dimensional analysis. IBM J. Res. Develop. **4** (1960), 349–356.

The normal method of dimensional analysis is replaced by a vectorial one in which, for example, Energy= ML^2T^{-2} becomes [1 2 $\underline{2}$]. [This approach has been anticipated by Drobot, Studia Math. **14** (1953), 84–99; MR **16**, 96.] The results obtained by the new method are identical with those of the old but transformations between systems are simplified.

{It appears that the proposed notation is ambiguous when a vector is deficient in any component. Thus the example quoted in the paper: Acceleration=[1 $\underline{2}$] might mean ML^{-2} or the true value LT^{-2}.}

A. D. Booth (London)

5152:

Atanasiu, Mihail. Sur certaines relations vectorielles utilisées dans la cinématique. Bul. Inst. Politehn. București **20** (1958), no. 4, 31–36. (Russian, English and German summaries)

Author's summary: "En utilisant dans une forme générale des systèmes de référence mobiles, l'auteur se propose de démontrer que les relations de base de la cinématique du mouvement relatif du point et du solide rigide constituent un cas particulier d'une règle de dérivation de la somme d'un nombre de vecteurs fonction d'un paramètre scalaire."

5153:

Stoenesco, Alexandre; Atanasiu, Mihail. Une généralisation de la loi des aires. Bul. Inst. Politehn. București **20** (1958), no. 3, 37–44. (Russian, English and German summaries)

A generalization of Kepler's second law (law of areas) for the central force motion is given for the case when the "central point" is in motion.

R. M. Evan-Iwanowski (Syracuse, N.Y.)

5154:

Mendes, Marcel. Sur les groupes de transformations canoniques. J. Math. Pures Appl. (9) **39** (1960), 165–171.

A transformation from $2n$ variables p_i, q_i to $2n$ new variables P_i, Q_i which involves t and r parameters a_j is given. For all the $a_j=0$, the equations yield the canonical conjugates in a Hamiltonian system. A condition in terms of the form of the infinitesimal transformations is obtained for P_i and Q_i to be canonical conjugates for all values of the a_j. *P. Franklin* (Cambridge, Mass.)

5155:

Barany, Paul. On a simultaneous collision of a system of bodies. Bul. Inst. Politehn. București **20** (1958), no. 4, 37–44. (Russian, French and German summaries)

Simultaneous collision is defined as one in which the compression period ends at the same time for all the bodies. A compatibility condition for this expresses the fact that the bodies must separate after the impact. For any initial velocities such a simultaneous collision is shown to be possible if all the coefficients of restitution are equal. *P. Franklin* (Cambridge, Mass.)

5156:

Košlyakov, V. N. The theory of a gyrocompass. Prikl. Mat. Meh. **23** (1959), 810–817 (Russian); translated as J. Appl. Math. Mech. **23**, 1164–1173.

Es werden Lösungen für die Bewegungsgleichungen eines Zwei-Kreisel-Kompasses untersucht, der nicht der Abstimmbedingung für den Raumkompaß von Geckeler genügt. Die ungestörten Bewegungen des Systems werden besonders für den Fall behandelt, daß sich der Kompaß auf einem Schiff befindet, das in hohen geographischen Breiten operiert. Dabei ist der Einfluß der Eigenbewegung des Schiffes von gleicher Größenordnung wie der Einfluß der Erddrehung. Es ist daher zu erwarten—und wird vom Verfasser bestätigt—, daß bei bestimmten Operationen des Schiffes instabile Bewegungen des Kompasses möglich sind. *K. Magnus* (Stuttgart)

5157:

Roitenberg, Ya. N. The accelerated placing of a gyroscopic compass in a meridian. Prikl. Mat. Meh. **23** (1959), 961–963 (Russian); translated as J. Appl. Math. Mech. **23**, 1370–1374.

Kreiselkompasse, die nach dem Schulerschen Prinzip auf eine Schwingungszeit von 84 Minuten abgestimmt sind, brauchen zum Einschwingen in die Nordrichtung etwa fünf Stunden. Der Verfasser untersucht die Möglichkeit, ein schnelleres Einschwingens dadurch zu erzwingen, daß zusätzliche Kräfte $Q(t)$ angewendet werden, die in die Gleichung für die Elevation eingehen. Die linearisierten Bewegungsgleichungen werden durch Vernachlässigen der Beschleunigungsglieder vereinfacht und gelöst. In die Lösung geht ein Integral ein, das wesentlich durch die Zeitabhängigkeit der Zusatzkräfte Q bestimmt ist. Durch Zerlegen der Funktion $Q(t)$ in drei Zeitintervalle, in denen Q jeweils konstant bleibt, läßt sich das Integral auswerten, und es lassen sich Forderungen für die drei Werte von Q ableiten. Ein Beispiel wird durchgerechnet.

K. Magnus (Stuttgart)

5158:

Četaev, N. G. On the stability of rough systems. Prikl. Mat. Meh. **24** (1960), 20–22 (Russian); translated as J. Appl. Math. Mech. **24**, 23–26.

Rough systems are non-linear systems for which the problems of stability can be solved by fairly simple approximate methods. This article deals with such a system for which the problem of the stability of motion reduces to the consideration of linear equations with constant coefficients. A sphere is determined in which the points of the disturbed motion will lie under certain fixed

initial conditions. The method of Liapunov is used for the derivation of stability conditions.
H. P. Thielman (Oxnard, Calif.)

5159:

Četaev, N. G. On certain questions related to the problem of the stability of unsteady motion. Prikl. Mat. Meh. **24** (1960), 6–19 (Russian); translated as J. Appl. Math. Mech. **24**, 5–22.

After some introductory remarks on the importance of Liapunov's method to the solution of important engineering problems on stability of motion, the author establishes a type of converse to a well-known theorem of Liapunov. He gives sufficient conditions for which undisturbed motions will be unstable, and deals with upper and lower bounds of the characteristic numbers of a system of linear differential equations. *H. P. Thielman* (Oxnard, Calif.)

5160:

Roža, P. [Rózsa, P.] On an application of latticed matrices in the mechanics of corpuscular systems. Uspehi Mat. Nauk **14** (1959), no. 4 (88), 207–211. (Russian)

By means of latticed (partitioned) matrices, the problem of oscillations of a two-dimensional corpuscular system (rectangular membrane) is solved. It is demonstrated that if the frequency ω of forced oscillations acting on any of the particles of the rectangular membrane satisfies the condition $\omega \geqq (2/T)\sqrt{2}$, where T is a particular constant related to the membrane considered, the periodic solutions of the differential equation

$$\ddot{\mathbf{W}}+\frac{1}{T^2}\mathbf{G}\mathbf{W}=\frac{1}{T^2}\mathbf{f}\sin\omega t,$$

G being the latticed matrix while **W** and **f** have the usual meaning, determine oscillations such that the displacements of adjacent elastically connected particles are of the opposite sign. In this way the Routh results concerning the oscillations of a chain of particles connected by strings are generalized. *T. P. Anđelić* (Belgrade)

5161:

Gantmacher, F. R.; Krein, M. G. ★Oszillationsmatrizen, Oszillationskerne und kleine Schwingungen mechanischer Systeme. Wissenschaftliche Bearbeitung der deutschen Ausgabe: Alfred Stöhr. Mathematische Lehrbücher und Monographien, I. Abteilung, Bd. V. Akademie-Verlag, Berlin, 1960. x+359 pp. DM 42.00.

This work is a translation of the 2nd Russian edition [Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1950] which was reviewed in MR **14**, 178; the authors have, however, contributed certain changes to this translation, such as revising the introduction, bringing the bibliography up to date and improving details.

5162:

Bufler, H.; Kiessling, F. Zur homogenen Maschine mit Zusatzdrehmassen. Ing.-Arch. **29** (1960), 250–259.

STATISTICAL THERMODYNAMICS AND MECHANICS

See also 5110, 5114, 5135.

5163:

Jeans, James. ★An introduction to the kinetic theory of gases. Cambridge University Press, New York, 1959. vii+311 pp. Paperbound: $2.95.

Paperbound reprinting of the first edition [1940; MR **2**, 139].

5164:

Arthurs, A. M.; Dalgarno, A. The theory of scattering by a rigid rotator. Proc. Roy. Soc. London. Ser. A **256** (1960), 540–551.

Racah methods are used to reduce the expressions for the elastic scattering cross-section, and for cross-section averages which occur in the kinetic theory of diffusion and viscosity, for scattering of particles by a rigid rotator molecule. *J. M. Blatt* (Murray Hill, N.J.)

5165:

Arthurs, A. M.; Dalgarno, A. The mobilities of ions in molecular gases. Proc. Roy. Soc. London. Ser. A **256** (1960), 552–558.

The mobility of an ion in a diatomic molecular gas is shown to be temperature-dependent at low temperatures (unlike ions in atomic gases) due to angle-dependent terms in the ion-molecule interaction. A minimum in the mobility is predicted at very low temperatures.
J. M. Blatt (Murray Hill, N.J.)

5166:

Laranjeira, M. F. An elementary theory of thermal and pressure diffusion in gaseous binary and complex mixtures. I. General theory. Physica **26** (1960), 409–416.

A simplified mean free path treatment is given of thermal and pressure diffusion. Two kinds of mean free paths are introduced, their ratio depending on the hardness of the molecules. *S. Simons* (London)

5167:

Laranjeira, M. F. An elementary theory of thermal and pressure diffusion in gaseous binary and complex mixtures. II. Binary mixtures with experimental comparison. Physica **26** (1960), 417–430.

The theory of thermal and pressure diffusion developed in paper I of this series is applied to binary mixtures. It is shown that the inverse of the thermal diffusion factor is in general linearly dependent on concentrations. This agrees with experiment and the Chapman first approximation. *S. Simons* (London)

5168:

Laranjeira, M. F.; Kistemaker, J. Experimental and theoretical thermal diffusion factors in gaseous mixtures. III. Ternary mixtures. Physica **26** (1960), 431–439.

The theory of thermal diffusion developed in papers I and II of this series is compared with experiment for various gas mixtures. Satisfactory agreement is obtained.
S. Simons (London)

5169:
Wentworth, R. C.; MacDonald, W. M.; Singer, S. F. Lifetimes of trapped radiation belt particles determined by Coulomb scattering. Phys. Fluids **2** (1959), 499–509.

The Fokker-Planck treatment of Rosenbluth, MacDonald, and Judd [Phys. Rev. (2) **107** (1957), 1–6; MR **19**, 335] of particles trapped in a magnetic mirror and perturbed by small angle Coulomb scattering is modified to treat fast protons and electrons trapped in the earth's field at altitudes of several earth-radii. Lifetimes and spatial distributions are calculated.

A. Herzenberg (Manchester)

5170:
Herman, Robert; Rubin, Robert J. Model for vibrational relaxation of diatomic gases behind shock waves. Phys. Fluids **2** (1959), 547–550.

5171:
Sankar, R. The van der Waals interaction of particles. J. Indian Inst. Sci. **42** (1960), 17–22.

5172:
Bonč-Bruevič, V. L. Spectral representations of many-time thermal Green's functions. Dokl. Akad. Nauk SSSR **129** (1959), 529–532 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1275–1278.

The causal Green function for n local operators $A, B, \cdots$ is defined by

$$K_c(x, x', x'', \cdots) = \langle T\{A(x)B(x')C(x'')\cdots\}\rangle,$$

where $x=\{x_0, \lambda\}$ represents the time and other variables, T means the time ordering and the average is taken over a Gibbs ensemble. This depends on $n-1$ temporal arguments and is nonanalytic. It is shown that 2^{n-1} retarded and advanced functions K_i may be introduced, the Fourier transforms of which are analytic functions of $n-1$ variables $E_1, E_2, \cdots, E_{n-1}$ in a region of appropriate sign of the imaginary parts of the variables. The set of all K_i forms a single analytic function $\tilde{K}$. *R. Kubo* (Tokyo)

5173:
Vineyard, George H. Molecular distribution functions involving two times. Phys. Fluids **3** (1960), 339–345.

Molecular distribution functions in a classical fluid are defined which represent the states of subsets of the particles at two different time points. These are simple generalizations of Yvon-Kirkwood-Born-Green distribution functions and obey the well-known hierarchy of integro-differential equations. The generalization of Kirkwood's superposition approximation is discussed in a formal way without physical applications.

R. Kubo (Tokyo)

5174:
Lebowitz, J. L.; Frisch, H. L.; Helfand, E. Nonequilibrium distribution functions in a fluid. Phys. Fluids **3** (1960), 325–338.

Transport properties in a classical fluid are first investigated by means of a particular form of the Fokker-Planck equation. Then, variational principles are considered which do not require use of this equation. They predict that the one- and two-particle distribution functions maximize a sum of entropy plus the entropy production during a certain characteristic time. (Adapted from the authors' summary) *R. H. Kraichnan* (New York)

5175:
McLennan, James A., Jr. Statistical mechanics of transport in fluids. Phys. Fluids **3** (1960), 493–502.

The author's [Phys. Rev. (2) **115** (1959), 1405–1409; MR **21** #7634] previous treatment of nonequilibrium Gibbsian ensembles is applied to a fluid in interaction with external reservoirs. Transport relations are obtained which are nonlocal in space and time. Upon a suitable averaging, they reduce to local relations previously obtained by Green and Mori.

R. H. Kraichnan (New York)

5176:
Ikeda, Kazuyosi. Some remarks on the Born-Green-Rodriguez theory of condensation. Progr. Theoret. Phys. **23** (1960), 616–628.

This paper has the useful object of clarifying the effect of approximations introduced by Born and Green, and Rodriguez, on the rigorous statistical mechanics of condensing fluids. It is known that in the condensation region the rigorous theory, though numerically intractable, must yield the thermodynamic functions of stable mixtures of liquid and gas, rather than those of the supercooled gas or superheated liquid. The author points out that the effect of the approximate theory is to exclude inhomogeneous states of the fluids, enforcing a departure from the stable isotherm. He begins by acknowledging the presence of the 'weight' parameter ε in the approximate theory, but afterwards discards this parameter with the remark that it is not important analytically. He does not state that he is thereby discussing a quantitatively different approximation. *H. S. Green* (Adelaide)

5177:
Katsura, S.; Harumi, K. A note on the Born-Green linearized integral equation. Proc. Phys. Soc. **75** (1960), 826–832.

The authors compare an exact evaluation of the partition function for a one-dimensional fluid, due to Gürsey, with that derived by an approximate method. The approximation is similar to one used by Born and Green for real fluids, but differs from it by the omission of the 'weight' parameter ε. The approximation is therefore actually equivalent to one discussed by Montroll and Mayer. The present authors show that, while this approximation is good at low densities, it gives qualitatively different results at high densities and predicts a condensation phenomena which is spurious. They do not notice that, if the weight parameter had been retained, the approximation would have been much improved, and the spurious condensation would have been removed from the physical region. *H. S. Green* (Adelaide)

5178:
Rushbrooke, G. S. On the hyper-chain approximation in the theory of classical fluids. Physica **26** (1960), 259–265.

The equilibrium pair distribution function $g(r)$ of a

classical fluid with central forces can be expanded as a power series in the density ρ. The coefficient of ρ^n is the sum of a number of n-fold integrals, each representable by a connected graph (cluster diagram) with $n+2$ vertices, two of which are labelled 1 and 2. The hyper-chain approximation neglects graphs of all but two types: 'chains' and 'bundles'. The simplest bundle consists of just one link. A chain consists of any number of bundles in series between the vertices 1 and 2; a bundle consists of any number of chains in parallel.

An integral equation satisfied by $g(r)$ in this approximation is derived here. The linearized form of this equation is identical with the linearized Born-Green equation.

O. Penrose (London)

5179:

Kobayashi, Shigehiro. Thomas-Fermi-Dirac model of compressed ions. J. Phys. Soc. Japan **15** (1960), 1842–1844.

Author's summary: "The Thomas-Fermi-Dirac (TFD) method is extended anew to ions in the compressed state and the series expansion coefficients of its TFD function near the boundary are given."

5180:

Morkowski, Janusz. On the approach to equilibrium of spin waves system. I. Acta Phys. Polon. **19** (1960), 3–19.

5181:

Tyablikov, S. V.; Šikloš, T. Quantum theory of uniaxial anisotropic ferromagnetic crystals. Acta Phys. Acad. Sci. Hungar. **12**, 35–46 (1960). (Russian. English summary)

5182:

Rubin, Robert J. Statistical dynamics of simple cubic lattices. Model for the study of Brownian motion. J. Mathematical Phys. **1** (1960), 309–318.

An n-dimensional cubic lattice of oscillating mass particles is used as a mechanical model for Brownian motion. One of the particles is given a mass which is much greater than that of the remaining particles. The equation of motion of the heavy particle is solved in terms of the initial conditions of the system by the use of Laplace transforms. It is then assumed that the initial conditions for the light particles can be specified by statistical considerations. The autocorrelation function for the initial conditions of the heavy particle, with its position and velocity at a later instant, is found. Special consideration is given to the distribution function of the velocity of the heavy particle, which is computed in several cases. {Certain of the properties of the velocity distribution function seem to the reviewer to be dictated by the use of periodic boundary conditions for the lattice, and so can hardly be considered to be an intrinsic part of the physical model.}

E. L. Hill (Minneapolis, Minn.)

5183:

Kohlmayr, G. F. Die Greensche Funktion zum Integrodifferentialoperator der stationären Neutronentransporttheorie. Acta Phys. Austriaca **13** (1960), 300–314.

The author considers the inhomogeneous integro-differential equation

$$(1)\quad (\mathbf{\Omega}\cdot\nabla+\Sigma(v))\psi(\mathbf{r},\mathbf{v}) - \int d\mathbf{v}'[\Sigma_s(\mathbf{v}/\mathbf{v}')+\nu\Sigma_p(\mathbf{v}/\mathbf{v}')]\psi(\mathbf{r},\mathbf{v}') = S(\mathbf{r},\mathbf{v})$$

($\mathbf{\Omega}=\mathbf{v}/|v|$), subject to the boundary conditions

$$(2)\quad \psi(\mathbf{s},\mathbf{v}) = 0 \quad \text{for} \quad \mathbf{\Omega}\cdot d\mathbf{S} < 0,$$

governing the neutron distribution $\psi(\mathbf{r},\mathbf{v})$ in presence of sources $S(\mathbf{r},\mathbf{v})$. In (2) $\mathbf{s}$ is a point at the outer surface of the space region and $d\mathbf{S}$ is a vector having the direction of the outer normal to that surface. The associated eigenvalue problem may be taken as

$$(3)\quad (\mathbf{\Omega}\cdot\nabla+\Sigma(v))\psi_k(\mathbf{r},\mathbf{v}) - \int d\mathbf{v}'[\Sigma_s(\mathbf{v}/\mathbf{v}')+\nu_k\Sigma_p(\mathbf{v}/\mathbf{v}')]\psi_k(\mathbf{r},\mathbf{v}') = 0$$

subject to the same boundary conditions (2).

Taking it for granted that the eigenfunctions $\psi_k(\mathbf{r},\mathbf{v})$ of (3) would form a complete set, the author constructs the Green's functions for (1) in terms of the $\psi_k(\mathbf{r},\mathbf{v})$. It should be pointed out, however, that unless suitable restrictions are imposed upon $\Sigma_s(\mathbf{v}/\mathbf{v}')$ and $\Sigma_p(\mathbf{v}/\mathbf{v}')$, examples can be constructed where the $\psi_k(\mathbf{r},\mathbf{v})$ do not form a complete set. It should be pointed out also that, from the computational point of view, the determination of an adequate number of eigenvalues ν_k and eigenfunctions ψ_k of (3) is more troublesome than the direct numerical solution of (1).

B. Davison (Toronto)

5184:

Singwi, K. S. On the theory of the diffusion cooling of neutrons in a finite solid moderator assembly. Ark. Fys. **16**, 385–411 (1960).

The author considers the time constant for the decay of a pulse of slow neutrons in a finite block of a solid (polycrystalline) moderator. The analysis is based on the "diffusion approximation" (i.e., the angular distribution of neutrons is assumed to be linear in the directional cosines of neutron travel); while, in so far as the energy dependence is concerned, this is expanded in terms of associated Laguerre polynomials of order unity and degree n; and the solutions are considered in detail for the cases when two terms, and when three terms, are retained in this expansion (L.p.e.). The scattering law is at first taken quite general, and then specialised to the case of Debye model of solids, incoherent scattering approximation.

If two terms are retained in the L.p.e. the results coincide with those obtained previously by a different method [Singwi and Kothari, J. Nuclear Energy **8** (1958), 59–62] but the improvements obtained by retaining three (or more) terms in the L.p.e. represent an advance on that earlier method.

Numerical results are compared with available experimental data.

B. Davison (Toronto)

5185:

Guth, E.; Inönü, E. Random-walk interpretation and generalization of linear Boltzmann equations, particularly for neutron transport. Phys. Rev. (2) **118** (1960), 899–900.

If the Boltzmann linear integro-differential equation of the neutron transport theory is converted, by the usual means, into an integral equation, and then solved by means of Neumann-Liouville series, the nth term of this series gives the distribution of neutrons which have suffered exactly n collisions. The paper draws attention to this self-evident fact and some of its implications.
B. Davison (Toronto)

5186:

Shuler, Kurt E. Relaxation processes in multistate systems. Phys. Fluids **2** (1959), 442–448.

ELASTICITY, PLASTICITY

See also 5149, 5298.

5187:

Rivlin, R. S. Some topics in finite elasticity. Structural mechanics, pp. 169–198. Pergamon Press, New York, 1960.

The author presents a brief account of the theory of large elastic deformations using rectangular cartesian coordinates and cartesian tensor notation in the hope that "this will make the material presented accessible to a larger audience than would have been the case if one or other of the more general tensor formalisms had been employed". The effect of symmetries in the undeformed state of the material is discussed and special emphasis is placed on materials which are isotropic initially. Simple problems are solved for incompressible isotropic bodies using a general form of strain energy. The paper closes with brief accounts of second-order theory and of the theory of infinitesimal deformations superposed on large deformations.
A. E. Green (Newcastle-upon-Tyne)

5188:

Pelczynski, T. Über die Mohrsche Spannungshypothese. Wiss. Z. Hochsch. Schwermaschinenbau Magdeburg **3** (1959), 143–147. (English and Russian summaries)

5189:

Mohan, Madan. Stresses due to a concentrated force in an infinite elastic parabolic plate not containing the focus. Bull. Calcutta Math. Soc. **51** (1959), 66–72.

5190:

Theocaris, P. S. The stress distribution in a semi-infinite strip subjected to a concentrated load. J. Appl. Mech. **26** (1959), 401–406.

A semi-infinite strip is subjected to concentrated axial loading. The stress distribution is found by a Schwarz-Christoffel mapping of the strip onto a half-plane, by using the field of isostatics in the case when a normal load is applied at the boundary of the half-plane and by relieving residual stresses on the longitudinal edges of the strip. Comparison of numerical and experimental results from photoelastic tests shows good agreement.
H. Bückner (Madison, Wis.)

5191:

Kuršin, L. M. Mixed plane boundary value problem of the theory of elasticity for a quadrant. Prikl. Mat. Meh. **23** (1959), 981–984 (Russian); translated as J. Appl. Math. Mech. **23**, 1403–1408.

The stress system due to a concentrated force at an interior point of the quadrant $x>0$, $y>0$ is determined for the following boundary conditions: displacements zero for $y=0$, external loading zero for $x=0$. The method is to consider the half-plane $x>0$ with two symmetrically placed concentrated forces and a load $q(x)$ distributed along the x-axis. An integral equation for $q(x)$ is found from the condition of zero displacement on $y=0$. The nature of the singularity near the origin is investigated.
R. C. T. Smith (Armidale)

5192:

Weber, C. Halbebene mit periodisch gewelltem Rand und periodischen Einzelkräften und Momenten. Z. Angew. Math. Mech. **40** (1960), 14–21.

The methods used are similar to those in an earlier paper by the same author [same Z. **22** (1942), 29–33; MR **4**, 230], conformal mapping onto a half plane with straight boundary followed by use of Fourier series for the stress function.
R. C. T. Smith (Armidale)

5193:

Mossakovskiĭ, V. I. On contact rolling of elastic cylinders. Prikl. Mat. Meh. **23** (1959), 989–990 (Russian); translated as J. Appl. Math. Mech. **23**, 1417–1419.

The author investigates the problem of the rolling due to inertia of a wheel of one material on a rail of a different material.
L. E. Payne (Newcastle-upon-Tyne)

5194:

Popov, G. Ya. Bending of an unbounded plate supported by an elastic half-space with a modulus of elasticity varying with depth. Prikl. Mat. Meh. **23** (1959), 1095–1100 (Russian); translated as J. Appl. Math. Mech. **23**, 1566–1573.

"For a half-space with a modulus of elasticity changing with the depth according to the law $E=E_\nu z^\nu$, the vertical displacements $w(x, y)$ of the boundary points $z=0$ of the half-space and the normal stresses $p(x, y)$ on the plane $z=0$ are connected by the relation

$$w(x, y) = \frac{1}{\pi D_\nu}\iint_S \frac{p(\xi, \eta)\,d\xi\,d\eta}{\sqrt{[(x-\xi)^2+(y-\eta)^2]^{\nu+1}}}, \quad D_\nu = \frac{\alpha}{E_\nu}."$$

A thin plate supported by a half space with the above properties is subjected to a load

$$q(x, y) = \delta(x)\cos\lambda y,$$

where $\delta(x)$ is the Dirac function. The normal pressure $p(x, y)$ between plate and half-space is then of the form

$$p(x, y) = p_\lambda(x)\cos\lambda y.$$

Next $p_\lambda(x)$ is shown to satisfy an integro-differential equation which is solved by Fourier transforms. The limiting case $\lambda=0$ is considered in detail. It is mentioned that the deflection problem for a semi-infinite plate lying on a half-space of the above characteristics reduces to an easily solved integral equation of Wiener-Hopf type.
R. C. T. Smith (Armidale)

5195:

Pan, Chia-cheng. The analysis of rigid frames on continuous elastic foundation. Sci. Sinica **9** (1960), 253–274.

This useful paper uses modified slope-deflection equations or other conventional analyses together with tables for beams on continuous elastic foundations. The theory is presented, and some examples are given; tables form an appendix to the paper.

J. Heyman (Cambridge, England)

5196:

Crichlow, Walter J.; Haggenmacher, Gernot W. The analysis of redundant structures by the use of high-speed digital computers. J. Aerospace Sci. **27** (1960), 595–606, 614.

The paper discusses the general problem of elastic structural analysis by means of computers. The matrix approach is adopted, and attention is given to the choice of the minimum size redundant system. Examples are given of the analysis of aircraft structures.

J. Heyman (Cambridge, England)

5197a:

Abramyan, B. L. Torsion of circular cylindrical rods with longitudinal grooves of wedge-shaped form. Akad. Nauk Armyan. SSR. Dokl. **28** (1959), 109–116. (Russian. Armenian summary)

5197b:

Abramyan, B. L. On torsion of a circular cylindrical rod with longitudinal cavities. Akad. Nauk Armyan. SSR. Dokl. **28** (1959), 201–211. (Russian. Armenian summary)

5197c:

Abramyan, B. L.; Babloyan, A. A. Torsion of circular rods having a longitudinal thread or teeth and a central cavity. Akad. Nauk Armyan. SSR. Dokl. **29** (1959), 203–209. (Russian. Armenian summary)

In each of these problems the boundary of the cross-section consists of circular arcs with centre the origin and segments of radial lines so that polar coordinates are appropriate. Solutions in terms of Fourier series are given in considerable detail.

R. C. T. Smith (Armidale)

5198:

Gupta, D. P. Stresses due to diametral forces in tension on an eccentric hole of a circular disc. Z. Angew. Math. Mech. **40** (1960), 246–252. (German and Russian summaries)

Stresses in a circular disc with an eccentric circular hole are determined, the loading being equal and opposite tensile forces acting on the boundary of the hole along the line of centres. The outer edge of the disc is unloaded. The complete solution is found in bipolar coordinates, and the results are applicable in the limiting case to a semi-infinite plate with a circular hole loaded perpendicular to the boundary.

J. Heyman (Cambridge, England)

5199:

Szmodits, K. Determination of limits in solutions of plate problems. Acta Tech. Acad. Sci. Hungar. **29** (1960), 245–250. (German, French and Russian summaries)

The principal upper and lower bound theorem in the method of the hypercircle due to Prager and Synge [see for instance J. L. Synge, *The hypercircle in mathematical physics*, Cambridge Univ. Press, New York, 1957; MR **20** #4073; p. 117] is briefly discussed. As an application to the bending of rectangular plates the role of the two well-known approximations by Fourier series, or in general by series of orthogonal functions, one satisfying the differential equation, the other the boundary conditions, is illustrated in the scope of this theorem.

W. Schumann (Zürich)

5200:

Tamate, O. Transverse flexure of a semi-infinite thin plate containing an infinite row of circular holes. J. Appl. Mech. **26** (1959), 661–665.

The title problem is solved by first constructing a set of periodic biharmonic functions which satisfy the condition of zero bending moment and reduced shear force along the straight edge of the plate. Coefficients are determined from similar conditions along one hole boundary. The solution is obtained by a perturbation method in which λ (ratio of hole radius to distance of hole center to straight edge) is a small parameter. Convergence is better for large distance between holes.

W. T. Koiter (Delft)

5201:

Bassali, W. A. Bending of a singularly loaded thin circular annulus with free boundaries. J. Mech. Phys. Solids **8** (1960), 123–140.

The problem of a thin, circular annular plate with free edges loaded by any system of concentrated forces or couples is investigated, series solutions being presented for the complex potentials and deflections. Closed-form solutions for the limiting cases of no hole and infinite outer radius are then presented.

Three loading cases for the ring plate are investigated in detail: (a) two bending couples at ends of a diameter of a concentric circle; (b) two twisting couples at ends of a diameter of a concentric circle; (c) four forces at ends of two perpendicular diameters of a concentric circle.

Finally, the deflection is obtained by the method of images for a sectorial plate bounded by two free arcs of concentric circles and two simply supported radii, the plate being subjected to a concentrated loading at an arbitrary point.

H. D. Conway (Ithaca, N.Y.)

5202:

Csonka, P. On the stress-function of the circular cylindrical shell. Acta Tech. Acad. Sci. Hungar. **29** (1960), 87–98. (German, French and Russian summaries)

5203:

Csonka, P. Paraboloid shell of revolution over equilateral triangle basis. Acta Tech. Acad. Sci. Hungar. **29** (1960), 313–332. (German, French and Russian summaries)

The membrane problem of the title is investigated, the edges of the shell being supported by rigid beams or

arches lying in vertical planes. An approximate stress-function approach is adopted, the validity of the latter being illustrated by a numerical example with two cases of loading. *H. D. Conway* (Ithaca, N.Y.)

5204:

Holländer, E. F. The basic equations of the dynamics of the continuous distribution of dislocations. I. General theory. Czechoslovak J. Phys. **10** (1960), 409–418. (Russian summary)

This is the first of three closely connected papers in which the problem of a general dislocation dynamics has been attacked. As is well known, quickly moving dislocations exhibit properties similar to those of fast moving particles in the special theory of relativity: contraction of fields, speed limit (now the velocity of sound), etc. Here an important difficulty arises: In contrast to light, sound has two velocities even in an isotropic medium. This means that "relativistic invariance" in our case is not Lorentz invariance but an invariance of a more complicated kind. As is, however, stated by the author, Lorentz invariance may be considered as a first approximation for dislocation velocities which are small compared with the velocity c_t of transversal sound waves.

On this point the reviewer wishes to remark: The linear theory of elasticity gives c_t as limiting velocity for the glide motion of screw and edge dislocations by which the volume elements are sheared. For climbing dislocations which elongate or shorten the volume elements another limiting velocity is to be expected, perhaps the velocity of longitudinal waves c_l. It follows that a theory taking c_t alone as limiting velocity is restricted to the description of gliding dislocations. The reviewer will come back to this later.

Now, the theory is formulated as a classical theory with respect to c_t relativistic linear field theory starting from Hamilton's variation principle. The Lagrangian is chosen in a way that guarantees the correspondence to the known formulation of the linear static continuum theory of dislocations with the restriction that curl $\boldsymbol{\epsilon}=\boldsymbol{\alpha}$ is taken as geometrical basic equation instead of curl $(\boldsymbol{\epsilon}+\boldsymbol{\omega})=\boldsymbol{\alpha}$ ($\boldsymbol{\alpha}$=tensor of dislocation density). This means that only such arrangements of dislocations are considered which lead to elastic deformations ($\boldsymbol{\epsilon}$) but not to rotations ($\boldsymbol{\omega}$) of the geometrical structure of the continuum. In the reviewer's opinion the inclusion into the theory of this rotation part would make the theory much more involved and would require certain principal extensions. The Lagrangian is constructed from 4-dimensional dislocation current densities (considered as given) and from 4-dimensional potentials the static components of which are the stress functions of 1st order. The potentials have to fulfill some kind of Lorentz conditions. The introduction of the potentials guarantees the existence of a 4-dimensional stress tensor the divergence of which vanishes identically. The space components of this tensor are identical with the ordinary stress tensor.

The interpretation of the time and space-time components of the quantities used in the theory is given in the second paper which is reviewed below.

E. Kröner (Stuttgart)

5205:

Holländer, E. F. The basic equations of the dynamics of the continuous distribution of dislocations. II. Interpretation of general theory. Czechoslovak J. Phys. **10** (1960), 479–487. (Russian summary)

In this paper is given physical content to the formulae of paper I [see preceding review] by interpretation of the quantities involved. First of all the 4-dimensional stress tensor E^{ik} is taken in the form

$$(E^{ik}) = \begin{pmatrix} (\sigma^{\iota\kappa}), & -ic_t\rho v^{\iota} \\ -ic_t\rho v^{\kappa}, & -\tfrac{1}{3}\sigma \end{pmatrix},$$

where i, k run from 1 to 4 and ι, κ from 1 to 3. $\sigma^{\iota\kappa}$ is the ordinary 3-dimensional stress tensor, σ its trace, ρ the density, v^{ι} the (total) matter velocity (caused by the moving dislocations). With this choice of E^{ik} the space part of the equations $\partial_i E^{ik}=0$ gives the (linearized) elastic-hydrodynamic equations of motion $\partial_\iota\sigma^{\iota\kappa}-\partial(\rho v^{\kappa})/\partial t=0$ and the time part the hydrodynamic continuity equation $\partial_\iota(\rho v^{\iota})+\partial\rho/\partial t=0$, which seems to be rather satisfactory. In the last equation the relation $-\partial\sigma/\partial t=2c_t^2\partial\rho/\partial t$ is used. This relation only holds in this form if the Poisson number $\nu=0$. This and later facts show (which has not been stated by the author) that it is possible to formulate a theory which is relativistic with respect to c_t only if $\nu=0$. A continuum with $\nu=0$ is called "elastic vacuum" by the author. He first derives the formulae for the elastic vacuum which seem to be completely correct and satisfactory. After this he tries to generalize to the case $\nu\neq 0$. After what the reviewer has said in the review of paper I this does not seem to be promising in a theory which is relativistically invariant with respect to c_t only. So, one finds inconsistencies in the generalized theory, e.g., the author uses a 4-dimensional Hookeian tensor which is not isotropic in space-time.

Nevertheless, the papers are highly interesting and encouraging, they show the way to a new general formulation of continuum mechanics which should bring progress in this field similar to that brought into electrodynamics by the special theory of relativity. The close connection between dislocation theory and hydrodynamics—manifesting itself essentially by the velocity field being the space-time part of the 4-dimensional stress tensor—is analogous to the connection between electro- and magnetostatics. This seems to be an especially important result. The physical background is the fact that it is possible to describe the motion of fluids, at least to a high degree, by the motion of dislocations.

E. Kröner (Stuttgart)

5206:

Holländer, E. F. The basic equations of the dynamics of the continuous distribution of dislocations. III. Special problems. Czechoslovak J. Phys. **10** (1960), 551–560. (Russian summary)

In this paper III [for I and II see preceding reviews] some more special problems of dislocation dynamics are considered. Of importance seems to be the concept of "Ohm's law of dislocation theory" in which a 5th order tensor of dislocation conductivity is defined which connects the dislocation current with the stresses. Hereafter the field equations are written again in 3-dimensional form and also an integral form is given. At the end the analogous quantities and formulae of the relativistic electrodynamics and dislocation dynamics are compared in a long list. *E. Kröner* (Stuttgart)

5207:

Toupin, R. A.; Rivlin, R. S. Dimensional changes in crystals caused by dislocations. J. Mathematical Phys. **1** (1960), 8–15.

According to classical linear elasticity theory the average value of each of the infinitesimal strain components is zero if dislocations are introduced into an elastic material, a result not in accordance with experimental data on cold worked metals. The authors use second order non-linear elasticity theory to calculate changes in the average dimensions of bodies, either isotropic or anisotropic, resulting from the introduction of dislocations. An explicit relation is obtained between the resultant volume change, the stored energy and the pressure derivatives of the elastic moduli. *A. E. Green* (Newcastle-upon-Tyne)

5208:

Kupradze, V. D. On boundary problems in the theory of elasticity for piecewise-inhomogeneous bodies. Soobšč. Akad. Nauk Gruzin. SSR **22** (1959), 521–528. (Russian)

Let a body with elastic constants λ_a, μ_a occupy a region bounded by a closed surface S_a and embrace an inclusion with constants λ_i, μ_i, bounded by a closed surface S. The following problems are discussed: (II.1) vibration of a bounded body when the displacements on the boundary S_a are prescribed; (II.2) vibration when the boundary tractions are prescribed; (II.0.1) equilibrium with boundary conditions as in (II.1); (II.0.2) equilibrium with boundary conditions as in (II.2); (III.0.1) and (III.0.2) as problems (II.0.1) and (II.0.2) but involving a half-space. *J. Nowinski* (Austin, Tex.)

5209:

Kupradze, V. D. Theory of boundary problems for non-homogeneous elastic bodies. Fundamental theorem of equivalence. Soobšč. Akad. Nauk Gruzin. SSR **22** (1959), 401–408. (Russian)

Let an elastic medium B_a with Lamé constants λ_a, μ_a occupy the entire space except for a finite number of disjoint closed regions B_i filled with elastic media of different coefficients λ_i, μ_i. If a source of periodic vibrations acts at a point P_0 of space, the determination of the stress field at $P \in (B_a + \sum B_i)$ is called Problem I. It was shown in a previous paper by the author [same Soobšč. **22** (1959), 129–136; MR **21** #3967] that if Problem I has a solution in the class B of vectors permitting an application of the Betti theorem, then this solution is also a solution of the system

$$(1)\qquad \begin{aligned} \mu_i u(P) &= Lu(P) + \mu_a F \quad \text{for } P \in B_i, \\ \mu_a u(P) &= L'u(P) + \mu_a E \quad \text{for } P \in B_a, \end{aligned}$$

where u is a displacement, L and L' operators, and F, E displacement fields.

The present paper investigates an inverse proposition: the solution of system (1) of class B is a solution of Problem I. *J. Nowinski* (Austin, Tex.)

5210:

Kupradze, V. D. Boundary problems in the theory of elasticity for piecewise non-homogeneous bodies. Soobšč. Akad. Nauk Gruzin. SSR **22** (1959), 265–271.

Existence of solution of Problem I [cf. preceding review] is given under the assumption that Poisson's ratios for the media B_a and B_i are equal. Since this is approximately true for all real bodies, the previous theorem is approximately true for any medium. *J. Nowinski* (Austin, Tex.)

5211:

Olszak, W. Anisotropy, non-linearity, plasticity and rheology in the theory of structures. Acta Mech. Sinica **4** (1960), 14–22. (Chinese)

The paper discusses the importance of anisotropy, non-linearity, plasticity, and rheology in the theory of structures and reviews the many papers published in these fields since 1953 by the author and his coworkers. The author points out that results obtained on the basis of such considerations not only are more realistic in the engineering sense but also may be quite different from the corresponding results given by the classical linear theory of elasticity. Although most of the author's contributions were published in Polish, a survey paper in English has recently become available, which gives an account of some of the topics discussed by the author [W. Olszak and W. Urbanowski, in *Non-homogeneity in elasticity and plasticity* (Symposium, Warsaw, 1958), Pergamon, New York, 1959; MR **21** #6791]. *Yi-Yuan Yu* (Brooklyn, N.Y.)

5212:

Stojek, Zbyszko. On the application of the Hamilton principle to the derivation of the bending equations of beams taking into account shears. Rozprawy Inż. **8** (1960), 201–210. (Polish. Russian and English summaries)

5213:

Alexander, J. M. An approximate analysis of the collapse of thin cylindrical shells under axial loading. Quart. J. Mech. Appl. Math. **13** (1960), 10–15.

The shell is assumed to collapse in a "bellows" mode, and a plastic work equation determines the collapse load. The analysis is partly empirical, but gives good agreement with quoted experimental results.

J. Heyman (Cambridge, England)

5214:

Shields, J. H.; MacNeal, R. H. The solution of elastic stability problems with the electric analog computer. J. Appl. Mech. **26** (1959), 635–642.

The paper deals with problems of elastic stability, in particular with lateral motion of axially loaded beams and edge-loaded plates. A continuous elastic system is represented as an electrical network of discrete units into which continuous elastic properties have been lumped. This amounts to a finite difference approximation of a differential equation. Electrical analogues of the following are discussed: Buckling of a column with hinged ends; buckling of a column with clamped ends; buckling of a simple frame; buckling of a beam on an elastic foundation; buckling of a flat plate; buckling of a simple angle.

H. Bückner (Madison, Wis.)

5215:

Bijlaard, P. P.; Gallagher, R. H. Elastic instability of a cylindrical shell under arbitrary circumferential variation of axial stress. J. Aerospace Sci. **27** (1960), 854–858, 866.

This is one of several recent classical stability studies (using small displacement theory and assuming perfect elasticity and shape) of thin cylinders under varying axial compression. All agree that the maximum critical stress in pure bending is little more than in uniform compression, and the present paper also finds a maximum critical stress only eight percent higher when the stress varies twice as rapidly as in bending. The availability of computers makes such studies feasible, but their significance may be questioned in such cases where experimental buckling stresses are only a fraction of the classical values. Tests show a ratio of around 1.3 between maximum bending and uniform buckling stresses on the average, but since the scatter tends to be greater in bending the minimum values are not very far apart.

L. H. Donnell (Ann Arbor, Mich.)

5216:

Harris, Leonard A.; Auelmann, Richard R. Stability of flat, simply supported corrugated-core sandwich plates under combined loads. J. Aero/Space Sci. **27** (1960), 525–534.

5217:

Mansfield, E. H. On the buckling of an annular plate. Quart. J. Mech. Appl. Math. **13** (1960), 16–23.

Author's summary: "This paper considers the buckling of an infinite plate supported along two concentric circles and subjected to a uniform radial compression, or tension, along the inner circle. The solution is also applicable to a similarly loaded finite annular plate if there is a member of the requisite tensile stiffness supporting the outer circle. The effect of regularly spaced diametral supporting members is also investigated."

R. C. T. Smith (Armidale)

5218:

Ogurcov, K. I.; Burova, A. V. Intensity of direct longitudinal and transverse waves propagated along the boundary of a halfspace. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1958**, 157–164. (Russian)

5219:

Konenkov, Yu. K. Plate waves and flexural oscillations of a plate. Akust. Ž. **6** (1960), 57–64 (Russian); translated as Soviet Physics. Acoust. **6**, 52–59.

The paper considers the propagation of flexural waves in an infinite elastic strip when the edges of the strip are free, hinged or clamped. Solutions of the resulting frequency equations are presented in graphical form for certain cases.

G. Eason (Newcastle-upon-Tyne)

5220:

Ang, Dang Dinh. Transient motion of a line load on the surface of an elastic half-space. Quart. Appl. Math. **18** (1960/61), 251–256.

Using Laplace transform methods, explicit algebraic expressions are obtained for the stress fields of the following two dimensional elasto-dynamic problem.

The free surface ($y=0$) of an elastic half space is subject to a normal stress component $\tau_{yy}=-\delta(x-t/C_0)$ for time $t>0$; for $t<0$ the solid is undisturbed. The formulae are derived for subsonic velocities C_0 although it is stated that the method of analysis can be used for values of C_0 in excess of either or both the elastic sound velocities. No numerical results are given.

S. C. Hunter (Stanford, Calif.)

5221:

Lee, E. H. Viscoelastic stress analysis. Structural mechanics, pp. 456–482. Pergamon Press, New York, 1960.

The paper gives a summary of the application of the theory of linear visco-elasticity in stress analysis. The general problem is defined in the introduction. Then laws of linear visco-elasticity, model representations, and approximation methods are reviewed. Subsequently the general problem of determining stress and strain distributions for non-homogeneous stressing is discussed. Application of a Laplace transform method or an operator technique can remove the time dependence and reduce the viscoelastic problem to an elastic one in reduced variables. It is concluded that the theory provides a basis for the complete solution of stress analysis problems for linear viscoelastic materials, but that sufficient experimental data are still missing; among the latter are accurate measurements of viscoelastic material properties, temperature effects, and limits of linear behavior.

B. Gross (Rio de Janeiro)

5222:

Galin, M. P. Propagation of elasto-plastic flexure-shear waves in beams. Izv. Akad. Nauk SSSR. Otd. Tehn. Nauk. Meh. Mašinostr. **1959**, no. 2, 88–99. (Russian)

The paper is concerned with propagation of a deformation disturbance in a semi-infinite elastic plastic rod loaded by a shear force at its end. Equations of the deformation theory of plasticity are assumed to hold good for dynamic problems. An incompressible material is considered. Equations of motion are written in terms of the beam axis deflection and the rotation of its cross-section. Normality condition of the plane cross-section and the neutral axis is not assumed. Characteristics of the set of equations obtained are given and two velocities of elastic-plastic wave fronts are obtained. Conditions for shock waves are discussed. Relations for unloading are sketched.

A. Sawczuk (Warsaw)

5223:

Jeffreys, Harold. Faults in a material that hardens when it yields. Proc. Roy. Soc. London. Ser. A **252** (1959), 431–435.

The stress solution is obtained for the case of an elastic solid with a perfectly rigid elliptical flaw embedded. The greatest stress differences arise at or near the ends of the flaw, and are determined mainly by the greatest principal stress. The case of a perfectly rigid spherical flaw is also briefly discussed, and compared with results of G. I. Taylor [same Proc. **145** (1934), 1–17] for the case when the flaw consists of compressible but weak material.

The paper also records a method of deriving the

equations of plane strain analogous to a method used in elastic wave problems.

Petrological considerations are included.

K. E. Bullen (Sydney)

5224:

Ivlev, D. D. Theory of perfectly hardening media. Dokl. Akad. Nauk SSSR **130** (1960), 742–745 (Russian); translated as Soviet Physics. Dokl. **5**, 66–70.

A perfectly hardening material is defined as one which (a) will deform freely under zero stress so long as a certain function of strain rate is less than k, (b) will support any magnitude of stress subject to a potential-type stress-strain-rate law when this function equals k, and (c) will not tolerate deformation rates which would make this function greater than k. Mathematically, such a material is the inverse of a perfectly plastic one, the roles of stress and strain rate being essentially interchanged.

The author restricts himself to an incompressible material, $\varepsilon_1+\varepsilon_2+\varepsilon_3=0$. He suggests that the strain-rate function analogous to the Tresca stress hexagon for a perfectly plastic material is the strain-rate hexagon whose typical side is $\varepsilon_1-(\varepsilon_2+\varepsilon_3)/2=k$.

For the case of plane strain $\varepsilon_3=0$ it is shown that the strain-rate point must be at a vertex of the hexagon. The system of defining equations is shown to be hyperbolic, with orthogonal characteristics which are the same for both velocity and stress, and are mathematically similar to those of plane plastic strain. If the strain point is at a vertex in the general three-dimensional case, then the velocity and stress equations may again be uncoupled. Each set is again of hyperbolic type and the characteristic surfaces of the two sets are the same.

P. G. Hodge, Jr. (Chicago, Ill.)

5225:

Berman, I.; Hodge, P. G., Jr. A general theory of piecewise linear plasticity for initially anisotropic materials. Arch. Mech. Stos. **11** (1959), 513–540. (Polish and Russian summaries)

For problems which may be solved by means of the theory of elasticity, the modulus of elasticity determines whether isotropy may be assumed. In most structural materials, the modulus of elasticity may be considered isotropic. However, structural materials are generally not isotropic in their initial yield surface or in their plastic flow characteristics.

This paper presents a method for the solution of problems in plasticity that involve structures which may be anisotropic in their initial yield surface, in plastic flow, and in the change of yield surface which is caused by strain hardening. The application is limited to those cases where principal directions of stress and strain in each element coincide and remain fixed throughout the loading process.

The method of analysis presented is one of piecewise linear plasticity. A general theory wherein a large number of material constants may be incorporated is first developed. The flexibility of the general theory provides a method by which better approximations to actual material properties may be utilized. Also, the general theory includes previous piecewise linear theories as special cases.

The mathematical difficulties, which may be encountered in the use of the general theory, may be reduced for the solution of some problems by the use of equations based upon the principal shear stresses. Specific equations for this simplified theory are given in tabular form.

W. T. Koiter (Delft)

5226:

Lepik, Yu. R. [Lepik, Ü.]. Analyse des nachkritischen Stadiums eines elastisch-plastischen Druckstabes mit Berücksichtigung der sekundären plastischen Deformationen. Tartu Riikl. Ül. Toimetised **73** (1959), 168–180. (Russian. Estonian and German summaries)

Problems of plastic instability of bars have been discussed so far under a restrictive assumption that no regions of secondary plastic deformation (i.e., involving tensile stress) exist, although such regions appear already by small deflections. The present paper is confined to an investigation of the influence of such regions on the post-buckling behavior of bars. Governing equations are derived as well as a method of determination of maximum load. An example involving idealized double-tee beam is solved.

J. Nowinski (Austin, Tex.)

5227:

Lepik, Yu. R. [Lepik, Ü.]. Die Biegung eines elastisch-plastischen Stabes im Falle vorangehender Dehnung. Tartu Riikl. Ül. Toimetised **73** (1959), 157–167. (Russian. Estonian and German summaries)

A straight bar with doubly symmetric cross-section, prestressed by a tensile force T and acted upon by a transverse load $q(x)$, is investigated, the slope $w(x)$ being assumed very small ($w_x^2 \ll 1$). Two cases are discussed: (a) stress induced by T is less than the yield stress; (b) this stress is greater than the yield stress. Influence of variation of T with variation of q is also considered. Equations are given for computations of w in terms of T and q.

J. Nowinski (Austin, Tex.)

5228:

Lepik, Yu. R. [Lepik, Ü.]. Zur Analyse des nachkritischen Stadiums der Platten, die ihre Druckstabilität bei plastischen Deformationen verloren haben. Tartu Riikl. Ül. Toimetised **73** (1959), 181–192. (Russian. Estonian and German summaries)

Postbuckling behavior of plates in plastic range is considered, using the Shanley concept. On the basis of the theory of small elastic-plastic deformations, a simple method of determination of the tangent line at a branch point of the stress-strain diagram is given. Four examples are solved concerning rectangular and circular plates.

J. Nowinski (Austin, Tex.)

5229:

Arutyunyan, N. H.; Manukyan, M. M. Plastic torsion of a conical rod. Akad. Nauk Armyan. SSR. Dokl. **29** (1959), 9–16. (Russian. Armenian summary)

Torsion of a shaft of variable diameter is studied for the following octahedral stress-strain law:

$$\varepsilon_i = \sigma_i G^{-1}(1+\beta\sigma^{m-1}G^{1-m}),$$

where G, β and $m>1$ are material constants. The solution is sought in terms of a stress function u defined in a standard way for rods of variable diameter. A non-linear differential equation for u is obtained, solution of which is sought in the form $u=u_0+\beta u_1+\beta^2u_2+\cdots$, where u_0 is the solution

for linear elastic material. A recurrent set of linear differential equations for $u_0, u_1, u_2, \cdots$ is obtained. For the particular case of a conical shaft a power series expression for u_1 is given. It is assumed that $u = u_0 + \beta u_1$ sufficiently approximates the exact solution of the problem for small values of β ("small" nonlinearity).

A. Sawczuk (Warsaw)

5230:

Greenspon, Joshua E. An approximation to the deflections and strains in a uniformly loaded, clamped, rectangular panel subjected to very large plastic deformations. J. Aero/Space Sci. **27** (1960), 392–393.

5231:

Rimrott, F. P. J. On the plastic behavior of rotating cylinders. J. Appl. Mech. **27** (1960), 309–315.

Stress and strain distributions are calculated for thick-walled long cylinders, assuming a power relationship between true stress and true strain. A numerical example is given, and the plastic instability speed is calculated.

J. Heyman (Cambridge, England)

5232:

Hamzawi, Hashim H. Plastic analysis of tapered cantilever beams. Proc. Iraqi Sci. Soc. **3** (1959), 13–18. (Arabic summary)

For prismatic beams, plastic hinges must occur at joints or loading points. For tapered beams, this is not necessarily true, and the author discusses the formation of plastic hinges in tapered cantilevers.

J. Heyman (Cambridge, England)

5233:

Finnie, I. Stress analysis in the presence of creep. Appl. Mech. Rev. **13** (1960), 705–712.

5234:

Zvolinskiĭ, N. V. On the emission (radiation) of an elastic wave from a spherical explosion in the ground. Prikl. Mat. Meh. **24** (1960), 126–133 (Russian); translated as J. Appl. Math. Mech. **24**, 166–176.

The problem of calculating the ground motion produced by the detonation of a high explosive charge is one of considerable physical and mathematical complexity. In this paper it is assumed that conditions of radial symmetry apply, the analysis therefore being appropriate to the detonation of a deeply buried spherical charge. The uniform infinite medium of soil in which the charge is enclosed is treated as an elastic/rigid-plastic solid which satisfies the generalized Hooke's law in the elastic range and a plasticity condition derived from the hypothesis that the increment of plastic work performed in passing from an elastic-plastic state of stress to a neighbouring state is proportional to the increment of maximum shear stress. Under the assumed conditions this criterion is formally equivalent to the appropriate form of Coulomb's law of failure. It is assumed that the transition from the elastic to the plastic régime is accompanied by an instantaneous density increase, no further compression of the soil being permitted during plastic flow.

Following the initiation of the explosive charge at its centre the motion of the soil is described in four stages (no detailed account being taken of the motion of the gaseous explosion products). In the first stage a shock wave travels outwards from the cavity wall, compacting the material through which it passes. As the radius of the shock front increases the velocity of propagation decreases. Stage 2 begins when the shock velocity becomes equal to the velocity of compressional elastic waves in the soil. An elastic wave then breaks away from the shock front and travels ahead of it. The shock velocity continues to decrease and eventually the discontinuity in particle velocity across the shock front is reduced to zero and the shock wave disappears. Stage 3 then begins, the interface between the plastically and elastically deforming regions now being treated as a moving contact discontinuity at which elastic waves continue to be generated. Further plastic yielding takes place but the interface and the cavity wall are rapidly brought to rest. There follows Stage 4 in which the motion of the soil is due entirely to elastic waves.

Stages 1, 2, 3 of the motion are discussed in detail, the various assumptions and approximations being clearly stated. Since the compacted soil is assumed to be incompressible the time variable can be effectively removed from the analysis and the governing equations reduced to non-linear ordinary differential equations of the first order. Having derived these equations and stated the conditions which the solutions must satisfy, the author deduces the salient features of the motion without resorting to detailed computation.

This is a most interesting paper which goes some way beyond previously published work on explosions in solid media. [For a review of spherical wave propagation in elastic and elastic-plastic solids with particular reference to metals, see H. G. Hopkins, *Progress in solid mechanics*, Vol. 1, pp. 83–164, North-Holland, Amsterdam, 1960; MR **22** #3254.] The provision of numerical results and a detailed appraisal of the basic assumptions, particularly those regarding shock wave propagation, would be welcome further developments. The translated version of the paper contains a number of misprints and obscurities.

P. Chadwick (Sheffield)

5235:

Buckens, F. Thermo-élasticité et thermo-rheologie. Rev. Questions Sci. (5) **21** (1960), 410–439.

The article surveys the importance of thermal effects occurring in engineering and in nature. Thermal stresses arise due to the incompatibility of thermal strains produced by non-uniform temperature distributions, and because body force distributions vary with the uneven changes in density produced by the temperature variation. Also the Fourier heat conduction equation is modified by the addition of two terms, one involving the time rate of change of dilatation and the other depending on the energy loss due to internal friction in the material. In practice thermal effects result in initial stresses which may be produced unintentionally in the manufacturing process or deliberately to pre-stress a component. A general non-mathematical discussion is given of the occurrence of thermal, initial and residual stresses in engineering (machine parts, turbines, nuclear reactors, ships, high-speed flight, re-entry) and in nature (geotectonics, effect on propagation of seismic waves).

F. J. Lockett (Sevenoaks)

5236:

Sneddon, I. N.; Lockett, F. J. The steady-state thermoelastic problem for the elastic layer resting on a rigid foundation. Ann. Mat. Pura Appl. (4) **50** (1960), 309–317.

A solution in terms of Fourier integrals is deduced for the steady-state thermal stresses in an infinite elastic slab, which is free from loading and rests on a rigid, ideally smooth, foundation. The underlying temperature field corresponds to an arbitrarily prescribed surface temperature at the free face of the slab and to a given heat flux at the opposite face. Since the assumed boundary conditions preclude any separation of the slab from the foundation and the latter is required to be frictionless, the results obtained can be realized physically only if the reactive normal traction is purely compressive.

E. Sternberg (Providence, R.I.)

STRUCTURE OF MATTER

See also 5377, 5411, 5432, 5438.

5237:

Izuyama, Takeo. Collective excitations of electrons in degenerate bands. I. Spin waves in Stoner's model of ferromagnetism. Progr. Theoret. Phys. **23** (1960), 969–983.

Author's summary: "Spin waves in the collective electron model of ferromagnetism are derived in a completely similar manner to that adopted in deriving exciton waves in insulators. The internal motion of an electron-hole pair forming the spin wave with a long wavelength is shown to be localized in the ordinary space. The frequency of the spin wave with long wavelength coincides with the result obtained by K. Yosida and T. Kasuya in the case where all electron spins are pointed toward the same direction in the ground state. It is concluded generally that spin waves break down unless there is a sufficiently large difference between the number of the electrons with up spin and the number of those with down spin."

5238:

Rebane, T. K. A variational principle for the calculation of a correction, quadratic with respect to the magnetic field intensity, to the electron energy in a molecule. Ž. Èksper. Teoret. Fiz. **38** (1960), 963–965 (Russian. English summary); translated as Soviet Physics. JETP **11**, 694–695.

Author's summary: "A variational principle is formulated for the calculation of the diamagnetic correction to the ground-state energy of an electron in a molecule. The method is based on the variation of the gauge-transformation functions of the vector potential."

5239:

Kosevič, A. M.; Andreev, V. V. On the quantum analog of the collision integral for electrons in magnetic and electric fields. Ž. Èksper. Teoret. Fiz. **38** (1960), 882–888 (Russian. English summary); translated as Soviet Physics. JETP **11**, 637–641.

The theory of electrical conductivity of metals in the presence of a strong external magnetic field is discussed by quantum mechanical perturbation methods. It is found that a small electric field may have an important influence on the scattering of conduction electrons by impurities in the lattice.

E. L. Hill (Minneapolis, Minn.)

5240:

Coumes, A. Oscillations collectives d'un système de trous en présence de phonons. J. Phys. Radium **21** (1960), 229–232. (English summary)

Author's summary: "The screening effect of light particles surrounding heavy particles (the charge of which is of the opposite sign) can be characterized independently of the screened particles. The collective oscillations of holes are derived by analogy with phonons. The damping of these oscillations is shown to be very small."

5241:

Babuška, I.; Vitásek, E.; Kroupa, F. Some applications of the discrete Fourier transform to problems of crystal lattice deformation. I. Czechoslovak J. Phys. **10** (1960), 419–427. (Russian summary)

Just as the classical Fourier transform finds application in the solution of differential equations, the authors apply the theory of the discrete Fourier transform to the solution of a system of difference equations which describes the positions of atoms in a deformed crystal lattice. In another application a two-dimensional difference equation is reduced to a one-dimensional difference equation.

H. A. Hauptman (Washington, D.C.)

5242:

Babuška, I.; Vitásek, E.; Kroupa, F. Some applications of the discrete Fourier transform to problems of crystal lattice deformation. II. Czechoslovak J. Phys. **10** (1960), 488–504. (Russian summary)

The discrete Fourier transform [see preceding review] is used to calculate the deformation of a crystal lattice as a consequence of (1) the application of external forces, (2) the existence of vacancies (missing atoms), or (3) the insertion of an atomic layer (edge dislocation). Application is also made to the solution of the problem of the oscillation of a one-dimensional atomic chain.

H. A. Hauptman (Washington, D.C.)

FLUID MECHANICS, ACOUSTICS

See also 5134, 5164, 5362, 5370, 5497.

5243:

Jacob, Caius. ★Introduction mathématique à la mécanique des fluides. Préface de Henri Villat. Éditions de l'Académie de la République Populaire Roumaine, Bucharest; Gauthier-Villars, Paris; 1959. 1286 pp. Lei 66.50.

Les 230 pages qui constituent la première partie de l'ouvrage sont consacrées à un exposé classique des principaux problèmes de la théorie des fonctions harmoniques et de la représentation conforme, avec un long chapitre sur l'utilisation de la théorie des équations intégrales, le tout limité au cas de deux dimensions. Tout en étant de facture assez classique, cette partie contient plus de développements qu'il n'en est donné habituellement dans un cours de Mécanique des fluides. Le cas des domaines

multiplement connexes est traité en détail. De nombreux problèmes aux limites variés, utiles pour la Mécanique des fluides incompressibles a deux dimensions, sont discutés. Certaines questions sont même présentées avec un grand luxe de détails : ainsi le théorème de Fatou-Privaloff sur le comportement à la frontière de l'intégrale de Poisson est-il discuté sous des hypothèses moins restrictives que l'habituelle condition de Hölder.

Avec les 230 pages de la seconde partie, commence l'étude de la mécanique des fluides. Les équations du mouvement des fluides visqueux ou non visqueux, compressibles ou incompressibles, les théorèmes généraux, et quelques problèmes classiques d'écoulement sont exposés. En raison du caractère même de l'ouvrage, le lecteur ne doit pas s'attendre à y trouver de discussion serrée sur les fondements thermodynamiques. D'aucuns pourront toutefois regretter de rares imprécisions, telles que "Pour évaluer l'énergie interne du fluide, il faut chauffer la masse fluide en vase clos à volume constant". Les théorèmes généraux sur les efforts globaux exerçés sur un obstacle en mouvement dans un fluide sont examinés, ainsi que la reconstitution des vitesses à partir des tourbillons. Enfin cette partie s'achève avec un exposé des plus classiques sur les écoulements plans d'un fluide incompressible.

Les 190 pages de la troisième partie traitent des théories de la résistance ou, plus exactement, des efforts hydrodynamiques : sillages de Helmholtz, théorie de l'aile mince d'envergure infinie. La théorie des surfaces portantes de Prandtl est exposée, mais, assez curieusement, elle n'est pas rattachée aux méthodes de linéarisation précédemment évoquées, dans la cas de l'aile d'envergure infinie. Les progrès récents concernant l'aile à jet et les écoulements non permanents sont heureusement inclus dans ce tour d'horizon, bien que le rapporteur ait regretté de ne pas voir au moins quelques indications sur les mouvements oscillatoires.

Les 550 pages qui restent, sont consacrées, en deux parties, aux fluides compressibles : théories exactes et méthodes d'approximation. Les propriétés générales des écoulements supersoniques sont heureusement dérivées d'une étude préalable de l'équation de Monge-Ampère. Les applications de la méthode hodographique de Tchapliguine, spécialements jets et sillages, sont présentées avec le même luxe de détail qui a déja été signalé. En fait, sur plus d'un point, ainsi que dans d'autres parties de l'ouvrage, l'auteur trouve là l'occasion d'exposer certains de ses travaux. N'étant pas spécialiste, le rapporteur a jugé ardue la lecture de bien des pages de cette partie. Pour citer un exemple, était-il absolument nécessaire, dans ce traité, de faire dériver l'étude de la bifurcation d'un courant indéfini par un dièdre, d'un passage à la limite à partir du cas où le courant est limité? Sur les écoulement supersoniques, l'essentiel est dit, mais pourquoi faire figurer dans cette partie, où il n'est question que de solutions exactes, une allusion à la théorie de Friedrichs sur l'interaction entre l'onde simple associée à un profil et l'onde de choc de tête, alors que cela eût assurément mieux trouvé sa place en quelqu'une des 300 pages de la dernière partie, consacrées aux méthodes approchées?

Pour donner une idée de la fin de l'ouvrage, disons qu'après un exposé très complet des méthodes d'approximation en subsonique, une place très importante est donnée, avec quelque 80 pages, à la théorie des écoulements supersoniques coniques linéarisés, au détriment d'autres questions non moins importantes. On peut regretter de ne pas voir dégager, avec toute la netteté désirable, les règles de similitudes auxquelles conduit la théorie linéaire. L'ouvrage s'achève sur un chapitre consacré aux écoulements transsoniques, avec cette fois mention explicite de la règle de similitude.

En bref, avec ses 1.290 pages dont la répartition n'est assurément pas la plus judicieuse possible, cet ouvrage peut difficilement être conseillé comme ouvrage d'enseignement; mais il rendra d'incontestables services aux spécialistes en raison de l'abondance des sujets traités et aussi, ce qui est important, des nombreuses références précises qui renvoient à chaque instant aux mémoires originaux. *J. P. Guiraud* (Paris)

5244:

Stanyukovich, K. P. ★Unsteady motion of continuous media. Translation edited by Maurice Holt; literal translation by J. George Adashko. Pergamon Press, New York-London-Oxford-Paris, 1960. xiii + 745 pp. $15.00.

Translation, with the cooperation of the author, of *Neustanovivšiesya dviženiya splošnoi sredy* [Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1955; MR **18**, 440].

5245:

Birkhoff, Garrett; Fisher, Joseph. Do vortex sheets roll up? Rend. Circ. Mat. Palermo (2) **8** (1959), 77–90.

The stability of vortex sheets in inviscid flow, and their tendency to roll up or unroll, are discussed. The problem is formulated and examined in terms of Hamiltonian dynamical systems. Numerical calculations are described which indicate that discrete vortex arrays should exhibit a random behavior. Critical comparisons with other theories are presented. *P. R. Garabedian* (New York)

5246:

Molčanov, A. M. The bounded variation of continuous solutions of the hydrodynamic equations. Dokl. Akad. Nauk SSSR **129** (1959), 1257–1260 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1210–1213.

Derivation of estimates of the total variation in space of solutions of certain hyperbolic systems that include as a special case the equations of hydrodynamics. Integrating factors and entropy play an important role in the analysis. *P. R. Garabedian* (New York)

5247:

Iacob, Caius. Une extension du théorème du cercle. Com. Acad. R. P. Romîne **9** (1959), 759–762. (Romanian. Russian and French summaries)

Milne-Thomson proved [Proc. Cambridge Philos. Soc. **36** (1940), 246–247; MR **1**, 284] the following "circle theorem": If $f(z)$ is the complex potential of an irrotational, inviscid, incompressible, plane fluid flow without rigid boundaries and if $f(z)$ has no singularities in $|z| \leq a$, then the introduction of a circular cylinder $|z| = a$ changes the complex potential to $w(z) = f(z) + \bar{f}(a^2/\bar{z})$. Generalizing this result, the author sets himself the following problem: Denote by D_1, C, D_2 the inside, boundary and outside of $|z| = a$, respectively; let $F_j(z)$ $(j = 1, 2)$ be defined (up to

additive constants) in the whole plane, having singularities in D_j only and let p_j be given, non-vanishing, real constants; then one wants to find functions $f_j(z)$, satisfying the following conditions: (a) $f_j(z)-F_j(z)$ is regular in D_j; and (b) the $f_j(z)$ are continuous in D_j+C and on C they satisfy $\operatorname{Re}\{p_2f_2(z)-p_1f_1(x)\}=\operatorname{Im}\{f_2(z)-f_1(z)\}=0$. The author shows that if $p=p_1+p_2=0$, the problem has, in general, no solution, while for $p\neq 0$ he obtains the explicit formula

$$pf_j(z) = pF_j(z)+(p_k-p_j)\overline{F_j(a^2/\bar{z})}+2p_kF_k(z)$$

($j, k=1, 2; j\neq k$), valid in D_j+C. {Reviewer's remark: In view of the conditions (i) $F_j(z)$ has (arbitrary) singularities in D_j; (ii) $f_j(z)-F_j(z)$ has no singularities in D_j, it seems that the condition (iii) $f_j(z)$ continuous in D_j+C is misleading; this seems also to be borne out by the author's explicit formula.} *E. Grosswald* (Philadelphia, Pa.)

5248:

Gheorghiţă, Şt. I. Sur l'application du théorème du cercle. Com. Acad. R. P. Romîne **9** (1959), 887–891. (Romanian. Russian and French summaries)

It may easily be shown by use of Milne-Thomson's circle theorem [see preceding review] that if one succeeds in mapping an irrotational, inviscid, incompressible, plane fluid flow conformally onto the exterior of a circle, then the problem of determining the complex potential is virtually solved. In order to obtain the desired conformal mapping, the author suggests a "semi-inverse" method. This consists of a sequence of simple, conformal mappings of certain canonical regions, each depending on some variable parameters; these may be adjusted so that the final mapping approximates well that of the desired configuration onto the exterior of a circle.

E. Grosswald (Philadelphia, Pa.)

5249:

Eckart, Carl. Variation principles of hydrodynamics. Phys. Fluids **3** (1960), 421–427.

The equations governing the motion of an incompressible fluid and the adiabatic motion of a compressible one are derived from variational principles in which the particle paths are varied. Interrelations between Bernoulli's principle and the circulation theorem are discussed, as is the application of Clebsch's transformation to the integration of the vorticity theorem. The author makes use of the variable κ whose time derivative is equal to the temperature to formulate a quantity he calls the total circulation which is conserved during the motion. This variable, introduced by Helmholtz [*Wissenschaftliche Abhandlungen*, Vol. I, J. A. Barth, Leipzig, 1882; p. 248] enters into the variational principle given by the reviewer [Proc. Symposia Appl. Math., Vol. I, pp. 148–157, Amer. Math. Soc., New York, 1949; MR **11**, 222].

A. H. Taub (Urbana, Ill.)

5250:

Dolapčiev, B.; Sendov, Bl. Symmetrical flow around a circular cylinder with two vortices behind it. Trajectories of the vortices and drag of the cylinder. Dokl. Akad. Nauk SSSR **128** (1959), 53–56 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 962–965.

This is the well known Föppl problem whose solutions the authors complete by computing the trajectories of the vortices and the drag on the cylinder.

L. M. Milne-Thomson (Madison, Wis.)

5251:

Nigam, Lakshmi N. Constant shear flow past two circular cylinders. Z. Angew. Math. Phys. **10** (1959), 584–592. (German summary)

Two-dimensional motions of inviscid liquid past two circular cylinders are considered using bipolar coordinates. The stream function of the undisturbed flow is of the form $ax+bx^2$, where a, b are constants; the line of centres of the cylinders is either parallel to Ox or to Oy.

W. R. Dean (London)

5252:

Martensen, Erich. Berechnung der Druckverteilung an Gitterprofilen in ebener Potentialströmung mit einer Fredholmschen Integralgleichung. Arch. Rational Mech. Anal. **3**, 235–270 (1959).

Plane potential flow through a cascade of congruent profiles is considered. The flow field can be computed from the flow velocity on the profiles; in order to obtain that velocity, an integral equation of the Fredholm type is formulated; this equation generalizes Prager's equation for the single profile. The integral equation is shown to have exactly one solution in its homogeneous form, a constant factor disregarded. This permits one to prescribe the circulation around a representative profile; with the circulation prescribed the inhomogeneous equation has exactly one solution.—For numerical purposes the integral equation is solved by substituting for the integral a finite sum in accordance with the trapezoidal rule. Numerical results obtained by means of an electronic computer are presented.—The reviewer would like to add that the integral equation seems to be one of the best tools in order to compute plane potential flow through cascades. Prior to the author's publication and independently of the author the General Electric Company (Large Steam Turbine and Generator Department) adopted the same integral equation for flow computations; the results were very satisfactory. *H. Bückner* (Madison, Wis.)

5253:

Biryukov, E. A. The downwash of the flow behind the swept vortex of finite span for unsteady motion. Prikl. Mat. Meh. **23** (1959), 583–584 (Russian); translated as J. Appl. Math. Mech. **23**, 823–826.

S. M. Belocerkovskiĭ's treatment [Prikl. Mat. Meh. **19** (1955), 159–164; MR **16**, 1060] of unsteady incompressible motion behind a rectangular wing of finite span by lifting line theory is extended to wings with small angle of sweepback. The circulation of the bound vortex line is assumed to vary harmonically with time and the downwash is determined by the Biot-Savart law. A key word in the title has been wrongly translated: "non-laminar" should read "non-stationary" or "unsteady".

M. Holt (Berkeley, Calif.)

5254:

Stewartson, K. A note on lifting line theory. Quart. J. Mech. Appl. Math. **13** (1960), 49–56.

The author points out that the more familiar methods of approximate solution of Prandtl's lifting-line integro-

differential equation (e.g., Fourier series) become less accurate as the aspect ratio is increased, except for certain wings with rounded tips. Therefore he undertakes to find a solution for a semi-infinite wing of constant chord. To accomplish this, a somewhat more complicated integral equation involving a Bessel-function kernel

$$f(\alpha, y) = -\frac{\alpha}{\pi}\int_0^\infty f(\alpha, x)K_1(\alpha|x-y|)\,\mathrm{sgn}\,(x-y)\,dx \quad (y > 0)$$

is solved by Fourier-transform methods. The result is a definite-integral expression for the circulation; numerical values are tabulated. Expansions for large and small values of the spanwise coordinate are given.

For this solution the aerodynamics of rectangular wings of large aspect ratio A can be calculated. Lift and induced drag are given to $O(A^{-2})$. The induced drag coefficient is proportional to $C_L{}^2A^{-1}\ln A$, where C_L is the lift coefficient. *W. R. Sears* (Ithaca, N.Y.)

5255:

Некрасов, А. И. [Nekrasov, A. I.]. **★Теория крыла в нестационарном потоке. [Theory of unsteady flow past a wing].** Izdat. Akad. Nauk SSSR, Moscow-Leningrad, 1947. 258 pp. 13.50 r.

5256:

Grodzovskiĭ, G. L. Selfsimilar motions with hydraulic waves in shallow water. Prikl. Mat. Meh. **23** (1959), 1143–1145 (Russian); translated as J. Appl. Math. Mech. **23**, 1640–1643.

The note sketches very briefly some results obtained by applying Sedov's similarity assumptions to shallow-water motions with a radially converging or diverging hydraulic jump. *R. E. Meyer* (Providence, R.I.)

5257:

Borisova, È. P.; Koryavov, P. P.; Moiseev, N. N. Plane and axially symmetrical automodel (similarity) problems of penetration and of stream impact. Prikl. Mat. Meh. **23** (1959), 347–360 (Russian); translated as J. Appl. Math. Mech. **23**, 490–507.

Generalization of the water entry problem to a model concerning a cone which penetrates a conical region of incompressible fluid at a rate proportional to some power of the time. Similarity solutions are considered and the resistance is computed on the basis of an approximate expression for the free surface.

P. R. Garabedian (New York)

5258:

Troesch, B. Andreas. Free oscillations of a fluid in a container. Boundary problems in differential equations, pp. 279–299. Univ. of Wisconsin Press, Madison, Wis., 1960.

Computation of the free oscillations of incompressible fluid in various containers. An inverse method of solution of this eigenvalue problem is presented. A variational treatment is also outlined.

P. R. Garabedian (New York)

5259:

Cumberbatch, E. The impact of a water wedge on a wall. J. Fluid Mech. **7** (1960), 353–374.

Similarity study of the impact of a wedge of water on a plane wall. The problem is quite analogous to that of water entry of a wedge. Various linearizations are exploited to find an approximate solution.

P. R. Garabedian (New York)

5260:

Sretenskiĭ, L. N. On the theory of gaseous jets. Prikl. Mat. Meh. **23** (1959), 305–332 (Russian); translated as J. Appl. Math. Mech. **23**, 436–472.

Problem of a free jet of compressible, inviscid fluid impinging on two straight walls and then issuing from these walls as another free jet. New solutions of Chaplygin's equations in the hodograph plane are used to find the answer. Similar treatment of free streamline flow past a flat plate preceded by a finite wake, and other models.

P. R. Garabedian (New York)

5261:

Sretenskiĭ, L. N. Wave diffraction in the Cauchy-Poisson problem. Dokl. Akad. Nauk SSSR **129** (1959), 59–60 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1193–1194.

This is a brief note dealing with the diffraction of waves produced in deep water by a local disturbance of the surface. The screen is a rigid half-plane with vertical edge. The method is to replace the Bessel function in the formula

$$\phi = \frac{gS}{2\pi}\int_0^\infty \frac{\sin \sigma t}{\sigma} e^{kz}J_0(kR)k\,dk$$

for undiffracted waves by Sommerfeld's solution of the problem of diffraction of cylindrical sound waves by a half-plane. *E. T. Copson* (St. Andrews)

5262:

Ursell, F. On Kelvin's ship-wave pattern. J. Fluid Mech. **8** (1960), 418–431.

The wave pattern due to a moving source on the surface of a fluid is discussed. In particular the critical regions along the track of the source and near the limiting lines of Kelvin are investigated. Near the track the (linearized) free surface oscillates with indefinitely increasing amplitude and indefinitely decreasing wave-length. It is shown that near the critical lines the crest length increases as the cube root of the distance from the source. The results for the critical lines follow from the method of estimating integrals given by Chester, Friedman and Ursell [Proc. Cambridge Philos. Soc. **53** (1957), 599–611; MR **19**, 853].

R. C. MacCamy (Pittsburgh, Pa.)

5263:

Stewartson, K.; Howarth, L. On the flow past a quarter infinite plate using Oseen's equations. J. Fluid Mech. **7** (1960), 1–21.

The authors investigate, on the basis of Oseen's equations, the flow past a quarter plane with its leading edge normal to, and its side edge parallel to, a uniform incident stream. They obtain the asymptotic development of the flow up to terms of order $\nu^{1/2}$ for small kinematic viscosity ν.

The solution is constructed by matching the flows due to a potential flow associated with the uniform stream, a (primary) shear flow which restores the no-slip condition

on the plane, and an additional potential flow and (secondary) shear flow required to restore the original boundary conditions.

These separate flows are analyzed in a region bounded away from the vertex of the plane. The regions in which they separately contribute to the skin friction, or are negligible, are carefully delimited. The results of this analysis are compared to the boundary layer solution, and it is shown that the cross flow in the side region "cannot be expressed in terms of what would be regarded as natural boundary layer variables but involves quite separately the distance from the leading edge".

G. E. Latta (Stanford, Calif.)

5264:

Kanwal, Ram Prakash. Impulsive rotatory motion of a circular disk in a viscous fluid. Z. Angew. Math. Phys. **10** (1959), 552–557. (German summary)

The author claims "to give an exact analytic solution, amenable to numerical work, of the flow when an infinitesimally thin circular disk of radius r_0 is given an impulsive moment $G\delta(t)$". The "exact" solution is for the linearized Navier-Stokes equation with one special form for the impulse stress distribution over the disk. The author seems to imply, however, that the total impulse should determine the stress distribution. The paper contains many misprints and several incorrect implications.

G. Newell (Providence, R.I.)

5265:

Rosenblat, S. Flow between torsionally oscillating disks. J. Fluid Mech. **8** (1960), 388–399.

An earlier paper [S. Rosenblat, same J. **6** (1959), 206–220; MR **22** #437] dealt with the motion induced by a disc oscillating in its plane through a small angle in a viscous fluid. Here the problem is generalised to include the effect of a parallel disc which is either fixed or oscillating 180° out of phase. In both cases it is found that when ν is small the first order effects are confined to boundary layers near the moving discs; elsewhere the flow is second order but includes a steady component implying the existence of a radial pressure gradient. Some formal comparison is made with the theory of the flow between parallel discs rotating with constant angular velocities.

K. Stewartson (Durham)

5266:

Komarov, A. M. Application of a method of Galerkin type for investigation of the development of perturbed flow of a viscous fluid in a plane channel. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 2, 55–59. (Russian)

The author considers the problem of the development of a perturbed flow of a viscous fluid in a plane channel by means of application of a method of Galerkin type. The stream function in a two-dimensional flow is expressed in form of a series; next, a method of Galerkin type [called by the author the method of Kantorovič, from L. V. Kantorovič and V. I. Krylov, *Približennye metody vysšego analiza*, Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1950; MR **13**, 77] is used to find the upper bounds for integrals over the space-time volume involving the velocity squared, the sum of the squares of the time derivatives of the velocity components, etc. The expressions representing the bounds consist of the exponential functions and their integrals. The inequalities so obtained allow one to obtain the maximum range of change in the magnitudes of the velocity components. The actual method of obtaining a solution of the problem in question is not constructed; the proposition refers rather to an outline of the procedure. No numerical example is calculated.

M. Z. v. Krzywoblocki (E. Lansing, Mich.)

5267:

Maude, A. D. The viscosity of a suspension of spheres. J. Fluid Mech. **7** (1960), 230–236.

From a general consideration of the slow motion of spheres in a sheared viscous liquid it is deduced that Einstein's expression, $1+2.5c$, for the proportional increase in viscosity arising from the spheres in suspension should be replaced by $(1-2.5c)^{-1}$; c denotes the volume of spheres in unit volume of suspension.

W. R. Dean (London)

5268:

Slezkin, N. A. On the development of the flow of a viscous heat-conducting gas in a pipe. Prikl. Mat. Meh. **23** (1959), 333–346 (Russian); translated as J. Appl. Math. Mech. **23**, 473–489.

The author treats the problem of flow of a viscous gas through a circular pipe whose wall temperature decreases along the axis according to an exponential law and whose velocity at the entrance is uniform over the section. For the case where the pressure is constant over every section, a solution is obtained by the method of perturbation. Results show that for subsonic flow at the entrance, the pressure will first decrease to the ambient pressure and continue to decrease exponentially below the ambient pressure, while the velocity along the axis will increase exponentially.

Y. H. Kuo (Peking)

5269:

Meksyn, D. Retarded flow past a semi-infinite plane. Solution of a non-linear ordinary differential equation. Z. Angew. Math. Mech. **40** (1960), 229–235. (German and Russian summaries)

The differential equation

$$\frac{\partial^3 f}{\partial \eta^3}+f\frac{\partial^2 f}{\partial \eta^2}=\lambda\left[1-\left(\frac{\partial f}{\partial \eta}\right)^2\right]+2\xi\left[\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \eta\,\partial \xi}-\frac{\partial f}{\partial \xi}\frac{\partial^2 f}{\partial \eta^2}\right]$$

(λ = known function of ξ) with the boundary conditions $f=\partial f/\partial\eta=0$ at $\eta=0$, $\partial f/\partial\eta=1$ at $\eta=\infty$, is solved by assuming a solution in the form $f(\xi,\eta)=\sum_{n=2}^{\infty}a_n(\xi)\eta^n$ and expressing all coefficients a_n in terms of the first coefficient a_2. The author then obtains a non-linear differential equation for a_2 by transforming an integral representation for $1=\partial f/\partial\eta$ at $\eta=\infty$ into a divergent series, which in turn is summed using Euler's transformation. The calculations are purely formal; a numerical illustration is given.

P. Henrici (Los Angeles, Calif.)

5270:

Reid, W. H. The oscillations of a viscous liquid drop. Quart. Appl. Math. **18** (1960/61), 86–89.

It is shown by a clever introduction of dimensionless parameters that for arbitrary values of the viscosity the problem of the small oscillations of a liquid globe when

the restoring force is surface tension is identical to the problem for the self-gravitating globe.

R. C. DiPrima (Troy, N.Y.)

5271:

Kogan, M. N. On flows with large heat conductions. Dokl. Akad. Nauk SSSR **128** (1959), 488–490 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 974–976.

This is a discussion of gas flows with very small Prandtl number. The thermal layer in such cases is much thicker than the underlying viscous layer, and viscosity can be neglected in it. The equations for this inviscid thermal boundary layer are solved here for a case of uniform pressure, stream temperature, and wall temperature. In the viscous layer the temperature and density are constant, and Blasius' well-known solution applies. Heat is generated by dissipation here and is transferred to the thermal layer and the wall; the transfer to the wall is determined by subtraction. The wall temperature for the insulated-wall case is determined; it is given by the same formula as for incompressible flow [Morgan, Pipkin, and Warner, J. Aero. Sci. **25** (1958), 173–180; MR **19**, 1119].

If the thermal conductivity k becomes very large, however, the two-layer concept fails; in the limit $k \to \infty$ the flow becomes isothermal. For large k the temperature obeys Laplace's equation, but at large distances from a body there arises a paradox analogous to Stokes' paradox for highly viscous flow. The author considers a case where stream and wall temperatures are only slightly different and writes a small-perturbation heat-flux equation. For the flat plate it is further simplified. For an arbitrary body it is proposed to calculate the solution as a perturbation of the isothermal solution.

W. R. Sears (Ithaca, N.Y.)

5272:

Rott, Nicholas; Rosenzweig, Martin L. On the response of the laminar boundary layer to small fluctuations of the free-stream velocity. J. Aerospace Sci. **27** (1960), 741–747, 787.

From the authors' summary: "The linearized treatment of small time-dependent disturbances of a laminar boundary layer, initiated by Lighthill, is extended in several ways. In particular, the high-frequency expansion is continued beyond the leading (Stokes) term. Several interesting questions of 'joining' occur, which are discussed but left unresolved. In addition, a practical method for obtaining the response to the laminar boundary layer to an impulsive change in velocity is presented. The methods are applied to the case in which the basic steady flow belongs to the Falkner and Skan family of similarity solutions."

The paper is written in review form with extensive reference to detailed solutions in papers not all of which have apparently been published yet.

K. Stewartson (Durham)

5273:

Watson, J. Three-dimensional disturbances in flow between parallel planes. Proc. Roy. Soc. London. Ser. A **254** (1960), 562–569.

In the study of the stability of parallel flows, Squire's results on the relationship between two- and three-dimensional disturbances have played an important role. These results are here generalized to Reynolds numbers above the minimum critical one. By introducing "stability surfaces" in the α-αR-β space, formed by rotating the usual two-dimensional stability curves in the α-αR plane about the αR-axis, it is shown that for a certain range of R above the critical, two-dimensional disturbances are the most unstable. For larger values of R each basic flow must be treated separately but it seems likely that a similar result holds. If both α and R are specified, however, then it is possible that the most unstable disturbance is a three-dimensional one.

W. H. Reid (Providence, R.I.)

5274:

Scheidegger, A. E. On the stability of displacement fronts in porous media: a discussion of the Muskat-Aronofsky model. Canad. J. Phys. **38** (1960), 153–162.

The author studies some instability problems related to viscous fluid flow in porous media, for the case when one of the fluids is more viscous than the other. The Muskat-Aronofsky model of displacement is employed in the analysis.

The process of the formation of "fingers" is also studied. However, no rigorous mathematical formulation is presented on account of formal difficulties. [Cf. #5324.]

K. Bhagwandin (Stockholm)

5275:

Richtmyer, Robert D. Taylor instability in shock acceleration of compressible fluids. Comm. Pure Appl. Math. **13** (1960), 297–319.

Let two fluids be separated by the interface $z=0$, with the lighter fluid in $z<0$. If a shock wave is incident on $z=0$ from $z<0$ the resulting acceleration, directed from the lighter to the heavier fluid, produces Taylor instability. Assuming the fluids to be compressible and the interface to have a small, sinusoidal displacement at $t=0$, the author formulates the initial-value problem for the subsequent growth of the interface. Numerical solutions are given for three different cases and lead to the conclusion "that if the initial compression of the interface and of the fluids is taken into account, the ultimate rate of growth of the [interface] agrees, to within 5 to 10%, with that given by the incompressible theory". In the latter approximation the acceleration of the interface is impulsive, and the amplitude of the sinusoidal interface grows linearly in time.

J. W. Miles (Los Angeles, Calif.)

5276:

Lin, C. C. On a theory of dispersion by continuous movements. Proc. Nat. Acad. Sci. U.S.A. **46** (1960), 566–570.

Taylor's theory of diffusion by continuous movements is applied to the problem of dispersion, i.e., the relative motion of two particles. The analysis leads to a Lagrangian parameter B which then appears in Richardson's law. This parameter B is of the same dimensions as ε, the rate of energy dissipation, and would, on Kolmogoroff's theory, be identified with it. The author suggests, however, that the ratio of these two parameters may well depend on the Reynolds number of the turbulence and that the determination of this ratio through experiment would provide a useful link between the Eulerian and Lagrangian descriptions.

W. H. Reid (Providence, R.I.)

5277:

Deissler, Robert G. On the decay of homogeneous turbulence before the final period. Phys. Fluids 1 (1958), 111–121.

The development of homogeneous turbulence is considered by forming the equation for the three-point triple-velocity correlation from the equations of motion, assuming that the terms in it involving quadruple correlations may be neglected and then combining it with the equation for the double-velocity correlation. Starting with an assumed initial form for the three-point triple correlation function the variations with time of the spectrum function, the turbulent energy and the spectral transfer function are calculated. The variation of turbulent intensity

$$\overline{u^2} = A(t-t_0)^{-5/2} + B(t-t_0)^{-7}$$

agrees fairly well with experimental measurements for times considerably before, as well as during, the final period of decay. In view of the analysis of Batchelor and Proudman, the reviewer feels some doubt concerning the validity of this way of improving the final period approximation. *A. A. Townsend* (Cambridge, England)

5278:

Deissler, Robert G. A theory of decaying homogeneous turbulence. Phys. Fluids 3 (1960), 176–187.

A previous paper [see preceding review] considered the decay of homogeneous turbulence by neglecting three-point quadruple-velocity correlations by comparison with three-point triple correlations, and this paper extends this approximation, suitable for small Reynolds numbers, by neglecting four-point quintuple correlations. When this is done, the equations for the double, triple and quadruple correlations, derived from the Navier-Stokes equations, form a determinate set and the development of a field of homogeneous turbulence may be predicted from the initial conditions. The effect of the improved approximation is to give an intensity decay of the form

$$\overline{u^2} = A(t-t_0)^{-5/2} + B(t-t_0)^{-7} + c(t-t_0)^{-19/2}$$

which is a good description of the observed decay of grid turbulence from the initial to the final period. It is interesting that, if the constants defining the initial conditions are obtained by fitting a spectrum curve to the experimental curve of Stewart and Townsend, the predicted decay of turbulent intensity is in good agreement with that observed at Reynolds numbers less than 1000.

A. A. Townsend (Cambridge, England)

5279:

Meecham, William C. Relation between time symmetry and reflection symmetry of turbulent fluids. Phys. Fluids 1 (1958), 408–410.

The time-symmetry of space-time velocity correlations of the kind

$$Q_{ij}(\mathbf{r}; T, t) = \overline{v_i(\mathbf{r}', t')v_j(\mathbf{r}'', t'')},$$

where $T = \frac{1}{2}(t'+t'')$, $t = t'-t''$, is investigated by assuming that mirror images may exist for all members of the ensemble used in the averaging process. This is permitted by the equations of motion. In this way, it is shown that the second-order correlation, Q_{ij}, and the fourth-order correlation have reflection symmetry in time, but not the third-order correlation. It is not necessary that the motion should be stationary in time.

A. A. Townsend (Cambridge, England)

5280:

Ohji, Michio. On the theory of homogeneous axisymmetric turbulence. III. Rep. Res. Inst. Appl. Mech. Kyushu Univ. 7 (1959), 259–278.

In two previous papers, the author [same Rep. 6 (1958), 63–83, 153–171; MR 21 #1095, 3187] has developed the theory of axisymmetric turbulence, considering for the most part the kinematics of the motion. In this paper, he discusses the dynamics of the motion, first outlining the arguments used by Batchelor and Proudman to show that the Loitsianski integral is not invariant and then developing the necessary modifications to the theory of the final period of decay and to Chandrasekhar's theory of axisymmetric turbulence. Various special cases are considered in some detail and in particular Chandrasekhar's special model of two non-interacting velocity fields, one transverse and one isotropic. The physical likelihood of such a motion is discussed and it is concluded that its existence is most unlikely unless the scales of the component motions are very different. The three papers are all clearly written and together they form a most useful, full and thoughtful account of the theory.

A. A. Townsend (Cambridge, England)

5281:

Pack, D. C. A note on the breakdown of continuity in the motion of a compressible fluid. J. Fluid Mech. 8 (1960), 103–108.

A simple, approximate estimate is given which indicates that, in plane, cylindrically or spherically symmetrical motion of inviscid gas, the occurrence of an infinite pressure gradient must be anticipated on any characteristic progressing outwards on which the motion is compressive. The ensuing shock may be too weak, however, to be detected in a numerical computation, and indeed, to be worth detecting. *R. E. Meyer* (Providence, R.I.)

5282:

Hanin, Meir. On Rayleigh's problem for compressible fluids. Quart. J. Mech. Appl. Math. 13 (1960), 184–198.

Howarth's [same J. 4 (1951), 157–169; MR 13, 179] theory of the motion of a viscous compressible gas engendered by the impulsive motion of an infinite flat plate is extended. The formal results are the same but whereas Howarth only calculated the pressure on the plate and the solution at large times, the author discusses the pressure and velocity distribution at arbitrary points and at arbitrary times. For this purpose some transformation of the integrals involved is necessary. One interesting feature of the paper is that the manner of formation of the sound pulse is clearly brought out.

K. Stewartson (Durham)

5283:

Cazacu, Mircea-Dimitrie. Mouvements unidimensionnels et non-permanents des fluides compressibles, dans le cas des petites variations de pression, avec des applications au coup de bélier. Bul. Inst. Politehn. Bucureşti 20 (1958), no. 3, 59–92. (Russian, English and German summaries)

The present work is devoted to a study of transition phenomena, which occur in a fluid medium which is perfectly elastic and barotropic, i.e., $p=p(\rho)$, due to the propagation of the pressure with the velocity of sound in a one-dimensional space. The velocity of the propagation of the pressure is assumed to be constant. The fundamental system of equations consists of two equations: conservation of momentum in one-dimensional motion and continuity equation. The author introduces a new independent, variable, $\tau=ct$, $c=$velocity of sound, and a polygenic function of a complex variable called a sonic function which satisfies the wave equation in question. The properties of this function are thoroughly examined. The sonic function must satisfy conditions which resemble Cauchy-Riemann equations and are of the form: $y_{,x}=h_{,\tau}$, $y_{,\tau}=h_{,x}$; or $y_{,x}=-h_{,\tau}$, $y_{,\tau}=-h_{,x}$ ($y=$pressure, $h=$velocity). The sonic function is used in the study of different methods which furnish the complex (and real) solutions of the wave equation for a compressible fluid. The author distinguishes two groups of those methods: indirect and direct. The domains of the flow are open or closed and in the latter case three different contours are considered. In the last part of the work the author applies this technique to the problem of a water-hammer phenomenon and outlines a possibility of solving more complicated problems which could not be solved otherwise.

M. Z. v. Krzywoblocki (E. Lansing, Mich.)

5284:

Thiruvenkatachar, V. R. Conditions characterising conical flows. Indian J. Math. **2**, 9–18 (1960).

The author considers the isentropic irrotational flow of a compressible fluid in which the direction of the velocity lies along straight lines passing through a common point. The magnitude of the velocity varies along each ray. In particular, the author studies the case when the flow is axially symmetric. Polar coordinates are introduced in any meridian plane and the rectangular components of velocity are represented by the product of a function of radial distance and a function of the polar angle. By use of the linearized potential flow equation, the forms of the above functions are explicitly determined in terms of the hypergeometric functions. However, in order to satisfy the equations of motion, it is shown that the velocity magnitude is constant along a ray or the flow is Taylor-Maccoll.

N. Coburn (Ann Arbor, Mich.)

5285:

Van Dyke, M. D. The paraboloid of revolution in subsonic flow. J. Math. Phys. **37** (1958), 38–51.

The author studies subsonic flow past a paraboloid of revolution at zero incidence by using the Janzen-Rayleigh expansion, i.e., by setting (in the usual notation)

$$\Phi = U[\phi_0+M\phi_1{}^2+(\gamma+1)M^4\phi_2+M^4\phi_3+\cdots]$$

and substituting in the equations of motion and boundary conditions. (The term $M^4\phi_3$ is omitted from consideration, for reasons explained.) The functions ϕ_0, ϕ_1, ϕ_2 are established explicitly at the surface. The results are studied numerically, and asymptotic formulas for the surface speed both near, and far away from the vertex are given. The author utilizes the above established Janzen-Rayleigh approximation to arrive at the second order small disturbance solution, with the aid of the similarity rule for linearized subsonic flow (as extended to second order). Another (direct) derivation is given for the small disturbance solution, the result agreeing with the one already found by means of the Janzen-Rayleigh approximation. Finally, the author derives a "nose correction rule" for correcting the first order slender body solution for the surface speed on a round-nosed body of revolution at zero angle of attack in subsonic flow.

W. Littman (Minneapolis, Minn.)

5286:

Ryžov, O. S.; Šefter, G. M. The approximate structure of one class of nonsteady transsonic flow. Dokl. Akad. Nauk SSSR **130** (1960), 276–279 (Russian); translated as Soviet Physics. Dokl. **5**, 1–4.

The author obtains a particular solution of a non-linear equation for unsteady transsonic flow. It is claimed, by analogy with a corresponding steady-state solution, that this represents flow with local supersonic zones in a nozzle with moving walls.

Despite an assertion by the author that the governing equation has been previously derived for flows varying rapidly with time, the reviewer believes that in fact it is only valid for slowly varying flows. It is essentially equation A1 of Miles [*The potential theory of unsteady supersonic flow*, Cambridge Univ. Press, New York, 1959; MR **21** #1105].

H. C. Levey (Perth)

5287:

Giese, John H. On the numerical representation of a singularity in axisymmetric supersonic flow. Symposium on the numerical treatment of partial differential equations with real characteristics: Proceedings of the Rome Symposium (28–29–30 January 1959) organized by the Provisional International Computation Centre, pp. 37–55. Libreria Eredi Virgilio Veschi, Rome, 1959. xii + 158 pp.

The accuracy of the tangent-cone approximation for calculating supersonic flow past pointed bodies of revolution is estimated in the case of the linearized problem. The formal solution of the cylindrical wave equation for the curved body at a given point is compared with that for the cone with the same local slope. No numerical results are given.

M. Holt (Berkeley, Calif.)

5288:

Ferri, A. Recent theoretical work in supersonic aerodynamics at the Polytechnic Institute of Brooklyn. Proceedings of the Conference on High-Speed Aeronautics, held January 20–22, 1955 at the Polytechnic Institute of Brooklyn, pp. 341–362.

Results of theoretical work on two problems are summarized. The first problem concerns the properties of the flow field about practical aerodynamic configurations (without recourse to linearization of the equations of motion). The method is to superimpose small disturbances on a basic nonlinearized flow field. Mixed-type (hyperbolic and elliptic) flows are considered. The author shows that under certain conditions transition from hyperbolic to elliptic flow requires two sonic lines. The second problem relates to interference problems with special emphasis on the high Mach number range. In the case of wing-body combinations, the flow field due to the body is simulated by a source distribution in a reference plane and interference effects are determined by linear methods. Integral

theorems, defining interference effects on lift and drag, are derived. These appear to be very useful for evaluating configurations designed for high Mach number flight. Viscous effects are neglected. *H. Mirels* (AMR **10**, 1171)

5289:

Bulah, B. M. Comments on the paper by A. Ferri, "Recent theoretical work in supersonic aerodynamics at the Polytechnic Institute of Brooklyn". Prikl. Mat. Meh. **23** (1959), 576–580 (Russian); translated as J. Appl. Math. Mech. **23**, 811–818.

The author comments on #5288 above.

5290:

Raymond, Joseph L. Piston theory applied to strong shocks and unsteady flow. J. Fluid Mech. 8 (1960), 509–513.

The author uses an empirical relation previously derived for shock pressure-coefficient in terms of $K_\infty (\equiv M_\infty \delta)$. Then he appears to assume the tangent wedge approximation, and that it is always valid to replace K_∞ (in steady flow) by w/a_∞, where w is the piston speed in the piston analogy. Hence he obtains a piston pressure versus piston-speed relation for $w > a_\infty$.

This is now applied to the case of a body with moving boundaries, $y = Y(x, t)$, so that $w = \partial Y/\partial t + U_\infty \partial Y/\partial x$. An example of a pitching biconvex aerofoil is presented. The reviewer is confused by the dual interpretation given to x in equation 15. *H. C. Levey* (Perth)

5291:

Černyĭ, G. G. Hypersonic flow past a thin cone with a blunted nose. Dokl. Akad. Nauk SSSR **115** (1957), 681–683. (Russian)

As in his treatment of the blunted wedge [same Dokl. **114** (1957), 721–724; MR **22** #2247] the author considers the analogous unsteady one-dimensional motion, the bluntness being simulated by a concentrated force. An approximation valid for values of the adiabatic exponent near unity leads to an ordinary nonlinear differential equation. The asymptotic solution for small distances downstream is the "blast-wave" solution of Cheng and Pallone [J. Aero. Sci. **23** (1956), 700–702] and Lees and Kubota [ibid. **24** (1957), 195–202]. A numerical solution for infinite Mach number shows qualitative agreement with American experiments. [A refined version of this theory appears in the author's subsequent comprehensive paper in Izv. Akad. Nauk SSSR Otd. Tehn. Nauk **1958**, no. 4, 54–66.] *M. D. Van Dyke* (Los Altos, Calif.)

5292:

Traugott, Stephen C. An approximate solution of the direct supersonic blunt-body problem for arbitrary axi-symmetric shapes. J. Aero/Space Sci. **27** (1960), 361–370.

Treatment of the detached shock problem by Belotserkovskii's method, which is based on an approximation to the partial differential equations of motion by cleverly chosen ordinary differential equations. New body shapes are discussed, and favorable comparisons with experimental data are described. *P. R. Garabedian* (New York)

5293:

Grodzovskiĭ, G. L.; Krašeninnikova, N. L. Self-similar motions of a gas with shock waves, spreading according to a power law into a gas at rest. Prikl. Mat. Meh. **23** (1959), 936–939 (Russian); translated as J. Appl. Math. Mech. **23**, 1328–1333.

Solutions of the problem of self-similar motion of a gas in which the shock wave obeys a power law $D = Ct^n$, where D is the shock velocity, t the time, and C a constant, are computed for a number of values of n. The results are used to describe hypersonic flow past a series of bodies of revolution and ducts. *M. Holt* (Berkeley, Calif.)

5294:

Bianco, E.; Cabannes, H.; Kuntzmann, J. Curvature of attached shock waves in steady axially symmetric flow. J. Fluid Mech. **7** (1960), 610–616.

Authors' summary: "An electronic computer has been employed to calculate the ratio between the initial radii of curvature of the attached shock wave and the body for an axially symmetrical body in a uniform supersonic stream. The results are obtained with four exact digits for more than 200 cases. They extend results obtained previously by means of numerical integration [H. Cabannes, Recherche Aéronautique No. 24 (1951), 17–23; MR **13**, 597]." *H. Polachek* (Washington, D.C.)

5295:

Kanwal, R. P. Propagation of curved shocks in pseudo-stationary three-dimensional gas flows. Illinois J. Math. **2** (1958), 129–136.

Cet article complète le travail Quart. Appl. Math. **16** (1958), 361–372 [MR **20** #6909], où se trouvent étudiés les gradients de vitesse et le rotationnel à la traversée d'une onde de choc dans le cas d'un écoulement et d'un choc stationnaire. L'auteur examine ici le cas pseudo-stationnaire: il s'agit d'un écoulement qui serait stationnaire si on prenait comme variables indépendantes $t^{-1}x_i$, x_i désignant les coordonnées cartésiennes. Le choc étant rapporté aux lignes de courbure, les calculs se font à l'aide de la géométrie infinitésimale. A titre d'application, l'auteur recense tous les cas où à un écoulement uniforme en amont du choc fait suite un écoulement irrotationnel. *P. Germain* (Paris)

5296:

Korobeĭnikov, V. P.; Ryazanov, E. V. A theory of linearized explosion problems including back pressure. Prikl. Mat. Meh. **23** (1959), 749–759 (Russian); translated as J. Appl. Math. Mech. **23**, 1066–1080.

In a gas with initial density $\rho_1(r) = Ar^{-\omega}$, where ω is a constant, consider the ν-dimensional plane, cylindrically-, or spherically-symmetrical flow due to a strong point explosion at $r=0$ at $t=0$. Particle velocity, density, and pressure (non-dimensionalized relative to their values behind the shock at time t) are expanded in series of the form $v/c = f(\lambda, q) = f_0(\lambda) + qf_1(\lambda) + \cdots$, etc., where $\lambda = r/r_2(t)$, $r_2(t)$ is shock radius, $c = dr_2/dt$, and $1/q = c^2/a_1^2$ is the square of the shock Mach number relative to the undisturbed gas. The functions f_0, etc., are chosen to be velocity, pressure, and density for a self-similar flow. The author exhibits a first integral for the first order system of ordinary differential equations for the coefficients of q.

For $(\gamma+1)\omega = 3\nu - 2 + (2-\nu)\gamma$ the system can be solved explicitly. Tables and graphs summarize computations of the explicit solutions.

J. H. Giese (Havre de Grace, Md.)

5297:

Pritchard, R. L. Mutual acoustic impedance between radiators in an infinite rigid plane. J. Acoust. Soc. Amer. **32** (1960), 730–737.

The mutual acoustic impedance of two identical circular disks (radius a, distance between centres d) is shown to be proportional to

$$\int_0^{\pi/2+i\infty} J_1^2(\alpha \sin\theta) J_0(\beta \sin\theta)(\sin\theta)^{-1}\, d\theta,$$

where $\alpha = ka$, $\beta = kd$ and $k = (2\pi)/(\text{wavelength})$. This integral is approximated in terms of elementary functions if ka and a/d are both small with respect to unity. An alternative series solution is obtained in terms of Bessel functions of integral and half-integral orders. As an application, the total acoustic loading of an array of circular discs is calculated, culminating in numerical results for an array of seven discs.

C. J. Bouwkamp (Eindhoven)

5298:

Payton, R. G. Transient interaction of an acoustic wave with a circular cylindrical elastic shell. J. Acoust. Soc. Amer. **32** (1960), 722–729.

Author's summary: "An infinitely long, circular cylindrical elastic shell is surrounded by an acoustic fluid. A plane pressure pulse, whose front is parallel to the axis of the shell, moves through the fluid, strikes the shell, and subsequently engulfs the shell.

"The circular shell is replaced by a fictitious Riemann surface which effectively allows the range of θ (the angular coordinate) to be extended from $-\infty$ to $+\infty$. Exact expressions are then found for the subsequent shell and fluid motion in the form of double integrals by the use of integral transform techniques. These integrals are evaluated asymptotically by the method of steepest descent to determine the early time motion of the shell and fluid. In particular, it is found that during this early motion the radial shell velocity and bending moment have a maximum, and the fluid pressure at the interface experiences a minimum."

F. G. Friedlander (Cambridge, England)

5299:

Genensky, Samuel M. A general theorem concerning the stability of a particular non-Newtonian fluid. Quart. Appl. Math. **18** (1960/61), 245–250.

The author considers the laminar flow of an incompressible non-Newtonian fluid in which the stress depends upon velocity and acceleration gradients and is a linear function of the appropriate rotationally-invariant tensors formed from them. He examines the stability of the motion under the influence of a two-dimensional disturbance and derives a stability criterion which depends upon the acceleration gradients.

J. E. Adkins (Providence, R.I.)

5300:

Reiner, M. Cross stresses in the laminar flow of liquids. Phys. Fluids **3** (1960), 427–432.

This is an experimental study of normal stresses generated in toluene by placing it between parallel plates, one of which rotates with constant angular velocity. It is argued that the observations cannot be predicted by the Navier-Stokes equations. The reasoning is that the analysis of Taylor and Saffman [J. Aero. Sci. **24** (1957), 553–562; MR **19**, 204] fails because it depends on compressibility, whereas the compressibility of toluene is negligible. Logically, this does not exclude the possibility that some different analysis would suffice. The author also argues that non-Newtonian effects might be expected since toluene may have a relaxation time large compared to that of air. The author concludes that some simpler theories of non-Newtonian fluids fail to explain the observations.

J. L. Ericksen (Baltimore, Md.)

5301:

Chandrasekhar, S. The stability of non-dissipative Couette flow in hydromagnetics. Proc. Nat. Acad. Sci. U.S.A. **46** (1960), 253–257.

The stabilizing effect of a constant axial magnetic field on an arbitrary angular velocity distribution is discussed in the case of zero viscosity and infinite conductivity. It is shown that an adverse gradient of angular velocity can always be stabilized by a sufficiently strong magnetic field the exact strength of which can be determined only from an explicit solution of a given problem. In the limit of zero magnetic field, it is found that Rayleigh's criterion for stability in a non-conducting fluid is not recovered. This last result is attributed to the fact that in a fluid of infinite conductivity the magnetic field is "frozen" in the fluid independently of its strength.

W. H. Reid (Providence, R.I.)

5302:

Ladyženskiĭ, M. D. Hypersonic flow past a body in magneto-hydrodynamics. Prikl. Mat. Meh. **23** (1959), 993–1005 (Russian); translated as J. Appl. Math. Mech. **23**, 1427–1443.

This paper considers the hypersonic flow of an electrically conducting fluid past an obstruction from within which a strong magnetic field is excited. The magnetic field acts on the fluid whose electrical conductivity greatly increases as a result of thermal ionization which occurs on transport of the fluid across the shock wave upstream from the obstruction. Such a situation is sufficient for a strong interaction between the electromagnetic effect and the hydrodynamic effect.

The flow past wedge-shaped and cone-shaped bodies is considered in detail, where the magnetic field vector is normal to the surface of the body. The solutions are based upon the assumption that the perturbed zone between the body and the shock wave is narrow.

It is shown, among other things, that under certain conditions the flow may separate from the wall of the body, which results in a decrease in the heat transfer to the body.

A. A. Mullin (Urbana, Ill.)

5303:

Johnson, J. L.; Oberman, C. R.; Kulsrud, R. M.; Frieman, E. A. Some stable hydromagnetic equilibria. Phys. Fluids **1** (1958), 281–296.

This paper is a calculation of some of the properties of

an almost uniform axial magnetic field in a perfectly conducting, infinitely long, almost cylindrical, almost pressureless fluid, given that the fluid is motionless and that outside it no electric currents can flow. Liapounoff's stability criterion is used to obtain some necessary conditions for the stability of such equilibrium systems. It is assumed, without proof, that these systems constitute a three-parameter analytic family, the three parameters measuring the deviation of the cylindroid from a cylinder, the ratio of fluid pressure to magnetic pressure, and the deviation of the magnetic field in the fluid from a vacuum field. *G. E. Backus* (La Jolla, Calif.)

5304:

Gotoh, Kanefusa. Stokes' flow of an electrically conducting fluid in a uniform magnetic field. J. Phys. Soc. Japan **15** (1960), 696–705.

This paper considers the flow of a viscous, incompressible, conducting fluid past an arbitrary three-dimensional body in a uniform applied magnetic field. The applied field is not necessarily parallel to the uniform applied flow field. The precursor and viscous wake are discussed and it is found that the distribution of charge shows the same structure. The flow past a sphere is discussed in detail. *H. P. Greenspan* (Cambridge, Mass.)

5305:

Chang, C. C.; Lundgren, T. S. Flow of an incompressible fluid in a hydromagnetic capacitor. Phys. Fluids **2** (1959), 627–632.

An incompressible conducting fluid is contained in a torus of rectangular cross-section. A uniform magnetic field is applied parallel to the polar axis; the fluid is set into motion upon applying a radial electric field. The basic equations, which are linear for this configuration, are solved in an approximate fashion and such overall properties as resistance, capacitance, transient response time, etc., are calculated. An equivalent electric circuit is determined. *H. P. Greenspan* (Cambridge, Mass.)

5306:

Drazin, P. G. Stability of parallel flow in a parallel magnetic field at small magnetic Reynolds numbers. J. Fluid Mech. **8** (1960), 130–142.

Continuing the work of Michael [Proc. Cambridge Philos. Soc. **49** (1953), 166–168; **51** (1955), 528–532; MR **14**, 596; **16**, 1174] and Stuart [Proc. Roy. Soc. London. Ser. A **221** (1954), 189–206; MR **15**, 907], the author attacks the title problem by generalizing known results for non-conducting fluids. He shows that "any given small wave disturbance can be stabilized by a sufficiently strong magnetic field if the Reynolds number is finite" but also shows that the jet and half-jet are unstable, by inviscid theory, to long-wave disturbances however strong the magnetic field. The author points out that the jet is made more unstable by the magnetic field. Analogously to the cases where viscosity is destabilizing, this is demonstrated to be due to the fact that the magnetic field alters phase relationships so that increased energy transfer from the basic flow to the disturbance more than offsets the magnetic energy dissipation. *L. A. Segel* (Troy, N.Y.)

5307:

Pacholczyk, Andrzej G. The magnetogravitational instability of an infinite compressible cylinder. The formulation of the local instability condition. Atti Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat. **94** (1959/60), 521–532. (Italian, Polish and Russian summaries)

The present paper deals with the local instability of a gravitating, infinite, compressible and isothermal cylinder in the presence of a magnetic field parallel to its axis. The medium is assumed to be non-viscous and of infinite electrical conductivity and the equilibrium values are assumed to depend on the distance from the axis only. Two cases have been considered: (1) when the magnetic field is uniform; (2) when the magnetic field is proportional to the density of the medium. In the second case the density is shown to be the solution of a second order non-linear differential equation, for which a series solution near the axis of the cylinder is obtained. The author has finally obtained the dispersion equations in the two cases, which he hopes to discuss in a further note. *F. C. Auluck* (Delhi)

5308:

Gross, R. A.; Chinitz, W.; Rivlin, T. J. Magnetohydrodynamic effects on exothermal waves. J. Aero/Space Sci. **27** (1960), 283–290.

The authors give the relations among the thermodynamic, mechanical, and electromagnetic variables at infinity on the two sides of a plane exothermic shock in an electrically conducting ideal gas containing a magnetic field parallel to the shock front. They could have reduced their shock to the classical non-hydromagnetic shock in a non-conducting gas with an equivalent pressure $\bar{p}=p+n\rho^2$ and an equivalent internal energy $\bar{U}=U+n\rho$, p, ρ, and U being pressure, density and internal energy of the conducting gas, and n being the magnetic energy density divided by ρ^2, a quantity continuous across the shock. The authors conclude with a brief summary of the electrical conductivities of real gases and the possibility of realizing their shocks experimentally. *G. E. Backus* (La Jolla, Calif.)

5309:

Kogan, M. N. Shock waves in magneto-gasdynamics. Prikl. Mat. Meh. **23** (1959), 557–563 (Russian); translated as J. Appl. Math. Mech. **23**, 784–792.

This paper discusses the hydromagnetic shock polars for all entropy-producing shocks in an ideal gas which is an electrical conductor on both sides of the shock. Some limiting cases are examined analytically, and the territory between them is covered by giving graphs of some undescribed numerical calculations. All cases could have been discussed analytically by means of a different choice of parameters. *G. E. Backus* (La Jolla, Calif.)

5310:

Šarikadze, D. V. A certain class of exact solutions of the equations of magneto-gasdynamics. Prikl. Mat. Meh. **23** (1959), 953 (Russian); translated as J. Appl. Math. Mech. **23**, 1356–1357.

The exact solutions exhibited here pertain to unsteady one-dimensional (rectilinear or plane radial) flows of an inviscid perfectly conducting gas with magnetic field perpendicular to the velocity vector. The pressure varies

with time only and the fluid acceleration is zero. (There is a misprint in the first paragraph of the translation: in the third case listed, m should be 1, not zero. The word "conical" in the first sentence is a misleading translation for "central".) *W. R. Sears* (Ithaca, N.Y.)

5311:

Nočevkina, I. I. On an approximation method in the analysis of plane rotational flow in magnetohydrodynamics. Dokl. Akad. Nauk SSSR **126** (1959), 1220–1223 (Russian); translated as Soviet Physics. Dokl. **4**, 549–553.

As is well known, in steady plane flow with a p, ρ-relation $p=A(\psi)\rho^n-B(\psi)$ where ψ is the stream function, the addition of a magnetic field normal to the plane of flow adds, in the case of infinite conductivity, the term grad $H^2/8\pi$ to the momentum equation; since $H/\rho=$ function of ψ, the magnetic body-force and the pressure add up to $P(\rho, \psi)$, which, in non-dimensional form, equals $Q(\psi)\rho^2+n^{-1}\rho^n-B^*(\psi)$ (not $n\rho^n$) and a corresponding (weak) Bernoulli equation. A hodograph-type transformation to the ρ, ϑ-plane leads to $\varphi_\rho=J\psi_\vartheta$, $\varphi_\vartheta=-K\psi_\rho$, with

$$J=-\lambda^{-1}[\rho^{-2}+(\rho v)^{-1}dv/d\rho]$$

and

$$K=[\rho(d\lambda/d\rho-\lambda v^{-1}\,dv/d\rho)]^{-1};$$

here the function λ was introduced by setting $\mathbf{v}=\lambda$ grad φ, with $\varphi=$const denoting the orthogonal trajectories of $\psi=$const.

For $n=2$, the speed v in J and K can be easily expressed in terms of ρ, and the function λ can be chosen so as to make $J=1$; $K(\rho)$ and $\lambda(\rho)$ are then quotients of polynomials. It is, however, possible to transform to an r, ϑ-plane by $r=f(\lambda)$ and, correspondingly, to consider K and λ as functions of r. The choice of $f(\lambda)$ provides, simultaneously, a second-order equation for $\psi(r, \vartheta)$ yielding a Bessel equation on separation, and a function $P(r, \psi)$ approximating the prescribed one in the vicinity of a given Mach number. (Work-in-progress report)

G. Kuerti (Cleveland, Ohio)

5312:

Tao, L. N. Magnetohydrodynamic effects on the formation of Couette flow. J. Aero/Space Sci. **27** (1960), 334–338.

An incompressible fluid with finite viscosity and electrical conductivity is confined between two infinite, parallel, electrically insulating plates. A magnetic field is maintained perpendicular to the plates. The upper plate suddenly starts to move parallel to itself at constant velocity. The author tries to find the subsequent motion of the fluid. Because he has not solved the steady problem correctly and because he has not given enough boundary conditions to determine the solution of the time-dependent problem, he fails. *G. E. Backus* (La Jolla, Calif.)

5313:

Rao, G. Teeka. Superposability of the equations of magneto-hydrodynamics. J. Math. Soc. Japan **12** (1960), 97–103.

Since the equations of magneto-hydrodynamics in an incompressible viscous electrically conducting fluid are non-linear, two solutions cannot be superposed in general. The author obtains two cases of cylindrical flow in which solutions are superposable. *E. T. Copson* (St. Andrews)

5314:

Arhipov, V. N. Influence of a magnetic field on boundary layer stability. Dokl. Akad. Nauk SSSR **129** (1959), 751–753 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1199–1201.

The stability of an incompressible, viscous, conducting boundary layer with a magnetic field $H(y)$ perpendicular to the plate is investigated using the Galerkin method. It is assumed that the basic flow is parallel and it is approximated by a sixth degree polynomial. The details of this approximation are not given. Using two terms in the approximating series for the eigenfunction reasonably satisfactory results for the critical Reynolds number are obtained in comparison with the results of the asymptotic solutions. This is quite surprising in view of the fact that it is well known that the Orr-Sommerfeld equation is very difficult to treat by approximate methods. Again, insufficient information is given to judge precisely what the author has done. The effect of the magnetic field is to increase the critical Reynolds number.

R. C. DiPrima (Troy, N.Y.)

5315:

Axford, W. I. The oscillating plate problem in magnetohydrodynamics. J. Fluid Mech. **8** (1960), 97–102.

The fluid is taken to be incompressible and there is a constant magnetic field perpendicular to the plate. It is shown that if $\sigma\nu\ll 1$ the motion consists of Alfvén waves together with a viscous boundary layer.

K. Stewartson (Durham)

5316:

Regirer, S. A. On convective motion of a conducting fluid between parallel vertical plates in a magnetic field. Ž. Èksper. Teoret. Fiz. **37** (1959), 212–216 (Russian); translated as Soviet Physics. JETP **10** (1960), 149–152.

Author's summary: "Stationary convective motion of a conducting fluid between vertical parallel plates in a magnetic field is considered. An exact solution of the magnetohydrodynamic equations is obtained for the case of a constant vertical temperature gradient."

H. P. Greenspan (Cambridge, Mass.)

5317:

Kleiman, Ya. Z. Special cases in the motion of two-component mixtures. Akust. Ž. **5** (1959), 301–313 (Russian); translated as Soviet Physics. Acoust. **5** (1960), 308–319.

Author's summary: "Certain cases of the nonstationary motion of a two-component mixture in the acoustic approximation are investigated, taking into account the friction between components, outflow of the mixture from a tube, propagation in the mixture of a disturbance arising at the interface between the media, disruptions in the mixture. As an illustration, the results of some numerical calculations are given for water-saturated sand."

5318:

Abasov, M. T.; Džalilov, K. N.; Kuliev, A. M. A three-dimensional problem of filtration of an elastic fluid on an

elastic stratum. Izv. Akad. Nauk Azerbaĭdžan. SSR. Ser. Fiz.-Teh. Him. Nauk **1959**, no. 1, 73–78. (Azerbaijani. Russian summary)

The authors solve the problem mentioned in the title by means of Laplace transform techniques. The infinite-series solution is expressed in terms of Bessel functions, sine and cosine integrals. For specified parameter variations, the authors put forth asymptotic expressions. No numerical results are presented.

K. Bhagwandin (Stockholm)

5319:

Sauvage de Saint-Marc, G.; Bouvard, M.; Ma, Min-Yuan. Pressions interstitielles dans les galeries en charge. Houille Blanche **15** (1960), 173–193. (English summary)

The authors study the elastic equilibrium of a thick-walled internally pressurized tube. Elasticity theory and Darcy-type flow field approximations are applied by the authors. {In the present reviewer's opinion, neither the derivation of the equations nor the ensuing solutions are realistic. A rigorous investigation of these types of problems will invariably lead one to a good deal of heavy mathematical analysis. The authors do not make a single reference to any work in this vast field.}

K. Bhagwandin (Stockholm)

5320:

Galin, L. A. Unsteady filtration of ground water in the case of a narrow drain. Prikl. Mat. Meh. **23** (1959), 789–791 (Russian); translated as J. Appl. Math. Mech. **23**, 1129–1133.

The author determines the position of the ground-water level in the case where there exists a narrow drain in the region occupied by the ground-water. The drain is not supposed to contain water, and its depth is comparatively small.

The problem is reduced to that of solving the Dirichlet problem for a cut half-plane. The solutions are obtained in terms of definite integrals over elementary functions. Numerical results are not presented.

K. Bhagwandin (Stockholm)

5321:

Lyaško, I. I. On a case of filtration from a canal. Dopovidi Akad. Nauk Ukraïn. RSR **1959**, 241–244. (Ukrainian. Russian and English summaries)

Author's summary: "The author derives formulae determining the upper estimate of the discharge of liquid in filtration from a canal with a horizontal aperture in an underground impermeable barrier of arbitrary shape.

"The estimate was obtained by the method of majorant regions. Applying these formulae it is easy to establish the region where it is most expedient to carry out geological investigations of the soil, and the region where it is unnecessary to conduct such investigations." The author applies conformal mapping techniques to obtain his solution in terms of elliptic integrals of the first kind. A few tabular entries are also presented.

K. Bhagwandin (Stockholm)

5322:

Gheorghitza, St. I. Motions with initial gradient. Quart. J. Mech. Appl. Math. **12** (1959), 280–286.

The author studies the motion of incompressible fluids in porous media with respect to the modulus of a certain critical pressure gradient. As is to be expected, Darcy's law is not valid for very small pressure-gradients. An equation is deduced which is applicable in the transition region; and the appropriate boundary-conditions are also stated. Rigorous solutions of these types of equations are, however, rather difficult to obtain. The solutions of some elementary cases are presented in closed forms.

K. Bhagwandin (Stockholm)

5323:

Aksent'ev, L. A. Sufficient conditions for univalency of the solution of the inverse problem of the theory of filtration. Uspehi Mat. Nauk **14** (1959), no. 4 (88), 133–140. (Russian)

5324:

Scheidegger, A. E. Growth instabilities on displacement fronts in porous media. Phys. Fluids **3** (1960), 94–104.

The author studies the penetration of a fluid into a porous medium which contains a more viscous fluid. He calculates flow potentials for stable displacement fronts. The differential equations in question are linearized. Fourier-analysis is applied to describe some of these processes. The process of fingering is also dealt with at some length. It is shown that under given external conditions, fingering is independent of the speed with which the displacement proceeds. Experimental findings testify this fact according to the author. [Cf. #5274.]

K. Bhagwandin (Stockholm)

5325:

Pryažinskaya, V. G. The problem of plane unsteady motion of ground waters. Prikl. Mat. Meh. **23** (1959), 954–957 (Russian); translated as J. Appl. Math. Mech. **23**, 1358–1364.

The author studies the problem mentioned in the title by means of the theory of singular integral equations. The solution of the physical problem is reduced to the solution of two coupled sets of non-linear integral equations. The author also states the appropriate boundary-conditions, as well as some theorems for the justification of some of her asymptotic estimates of the entailing functional expressions. She also states the necessary existence proofs. The physical problem is, however, not solved; neither does the author indicate the methods to be employed to this effect. *K. Bhagwandin* (Stockholm)

5326:

Pattle, R. E. Diffusion from an instantaneous point source with a concentration-dependent coefficient. Quart. J. Mech. Appl. Math. **12** (1959), 407–409.

The author obtains an expression for the concentration distribution produced by diffusion from an instantaneous point source in one, two, or three dimensions, when the diffusion coefficient varies as a positive power of the concentration. A graph is presented for the case of two-dimensional diffusion. *K. Bhagwandin* (Stockholm)

5327:

Weissberg, Harold L. Laminar flow in the entrance region of a porous pipe. Phys. Fluids **2** (1959), 510–516.

The author studies the motion of steady, incompressible, axially symmetric flow in a pipe with uniform wall-suction. The Navier-Stokes' equations of motion are solved for the case of high axial Reynolds' number. Graphical results related to the velocity profiles, velocity gradients at the wall of the pipe and pressure distributions are also presented. *K. Bhagwandin* (Stockholm)

5328:

Parsons, D. H. A note on one-dimensional diffusion. J. London Math. Soc. **34** (1959), 449–450.

The author studies one-dimensional diffusion from an initially sharp boundary between two semi-infinite columns of liquid. The problem is reduced to that of finding integrals of a certain first-order differential equation. Numerical results are not presented.

K. Bhagwandin (Stockholm)

5329:

Ting, Lu. On the mixing of two parallel streams. J. Math. Phys. **38** (1959/60), 153–165.

This paper concerns the question of indeterminacy in the problem of mixing of two parallel streams. It is known that the solution in such a problem has been obtained subject to only two boundary conditions and, hence, contains an arbitrary constant. The missing third condition is now given by the author and can be ascribed to the balance of pressures of the parallel streams.

Y. H. Kuo (Peking)

5330:

Kovitz, A. A.; Hoglund, R. F. Laminar parallel stream mixing with dissociation and recombination. Phys. Fluids **3** (1960), 436–443.

Using a slightly modified form of the Blasius variables, the atom continuity and energy equations are written in a form suitable for an attack on the two-dimensional laminar mixing problem. The flow velocities of the two streams, which begin to mix at $x=0$, are assumed constant throughout and a simple solution obtained for the enthalpy when the Lewis-Semenov number (Le) is unity. This solution is valid for arbitrary reaction rates and is a function of the Blasius variable $\psi^*/(x^*)^{1/2}$ only. (ψ^* and x^* are dimensionless stream function and streamwise coordinate respectively.)

Assuming a reaction rate which depends linearly on the difference between local actual and equilibrium atom mass fractions, it is found that the atom concentration equation can be separated into two ordinary differential equations. One of these can be reduced to the Hermite equation and solutions can then be found for any value of Le. The solution for temperature distribution in equilibrium flow is found for arbitrary Le and an approximate form suggests weak dependence on this parameter, provided it is near to unity. Frozen and equilibrium mixing do not differ greatly when $Le\simeq 1$, provided equilibrium exists outside the mixing region, but this would not be so if the streams were not in equilibrium outside the mixing region. *J. F. Clarke* (Cranfield)

OPTICS, ELECTROMAGNETIC THEORY, CIRCUITS

See also A4722, 5151, 5169, 5307, 5506.

5331:

Miyamoto, Kenro. On Gabor's expansion theorem. J. Opt. Soc. Amer. **50** (1960), 856–858.

Author's summary: "As a generalization for optics of the well-known sampling theorem of information theory, D. Gabor proposed an expansion theorem. It relates to the number of independent solutions of the wave equation in a region defined by the object and by the aperture of an optical system.

"A proof of this theorem presents formidable difficulties. In this paper, a proof relating to important cases is established, and a more accurate estimate for the number of the independent solutions in the general case is given."

S. Kullback (Washington, D.C.)

5332:

Sumi, Masao. Non-linear effects on electron-plasma oscillations. J. Phys. Soc. Japan **15** (1960), 1086–1093.

Author's summary: "Non-linear effects are investigated in the excitation of electron-plasma oscillations, by considering a non-linear term in the Boltzmann equation as a perturbation. With the growth of fundamental component, the second and the higher harmonics are generated as forced oscillations. The magnitudes of these components are estimated which give the criteria of validity of linear approximation. Finally the behaviours of excited waves in the limit of large amplitude are described."

5333:

Rand, S. Damping of the satellite wake in the ionosphere. Phys. Fluids **3** (1960), 588–599.

The author investigates the damping in a satellite wake moving supersonically through the ionosphere by studying the wake produced by an infinite line charge moving supersonically through a plasma. Supersonic flow means $MV^2/kT_i\gg 1$, where M and T_i are the ion mass and temperature, k is the Boltzmann constant and V is the velocity of the line charge with respect to the plasma. Maxwellian distribution functions in the ambient plasma for both electrons and ions are assumed. The linearized particle method used in the analysis is an extension of that used previously by the author [Phys. Fluids **3** (1960), 265–273; MR **22** #3374]. He finds that unless T_e is at least an order of magnitude greater than T_i the wake begins to damp after about a Mach number of Debye lengths behind the line charge. Also, after damping begins, the wake decreases quadratically with distance behind the charge, but as T_e/T_i increases above a value of about ten, the damping rapidly becomes unimportant. The author claims that many of his results are still valid if the line charge is replaced by a three-dimensional body simulating a satellite. With the exception of the peak value of the potential the results do not depend upon the shape of the body. Furthermore, it is shown that the damping is critical to the question of the observability of a satellite electrohydrodynamic wake in the ionosphere unless the electron temperature is of an order of magnitude greater than the ion temperature. *R. S. B. Ong* (Leiden)

5334:

Wyld, H. W., Jr. Radiation by plasma oscillations in a bounded plasma in a magnetic field. Phys. Fluids **3** (1960), 408–415.

The author investigates the possibility of radiation by plasma oscillations in a bounded plasma with a background magnetic field. Treating the radiation as a surface phenomenon taking place at the boundary of the plasma, he calculates it in the limit $(\omega_c/\omega_p)^{1/2} \ll 1$, where ω_c and ω_p are the cyclotron and plasma frequencies. Two different methods are used: (1) by means of a "Fresnel formula" for the ratio of the intensity of the electromagnetic wave transmitted through the plasma boundary to the intensity of the incident plasma wave; (2) by finding the decaying modes which satisfy a radiation condition at infinity. The ion mass is taken to be infinite, which is seemingly justified by the final result obtained. Also, the thermal velocity of the plasma is neglected in the equation of motion and boundary condition. The author justifies this by a brief order-of-magnitude analysis of the errors involved. The two methods described above yield the same result and the author arrives at an expression for the energy radiated per unit time per unit surface area of the plasma. He finds that for a thermal distribution of plasma oscillation the radiation due to plasma oscillations is small compared to the synchrotron radiation. *R. S. B. Ong* (Leiden)

5335:

Dodo, Tarô. Heating of a plasma by acoustic waves. J. Phys. Soc. Japan **15** (1960), 1292–1295.

Author's summary: "Generation of an acoustic wave and transformation of its energy into thermal energy are considered. We generate an acoustic wave, imposing an oscillating magnetic field to a cylindrical plasma in a static magnetic field. When the suitable frequency of oscillating field is chosen, an acoustic wave is excited at an intermediate state between adiabatic and isothermal states. In this situation acoustic energies are transformed into thermal energies through the irreversible flow of heat due to stress relaxation."

5336:

Kovrižnyh, L. M.; Ruhadze, A. A. Instability of longitudinal oscillations of an electron-ion plasma. Ž. Èksper. Teoret. Fiz. **38** (1960), 850–853 (Russian. English summary); translated as Soviet Physics. JETP **11**, 615–616.

Authors' summary: "The problem of the instability of longitudinal oscillations of low temperature electron-ion plasma is discussed. In an isotropic plasma the oscillations are always damped, while in an anisotropic one the ion motion may lead to the appearance of solutions that increase with time, i.e., to instability."

5337:

Buchsbaum, S. J.; Mower, Lyman; Brown, Sanborn C. Interaction between cold plasmas and guided electromagnetic waves. Phys. Fluids **3** (1960), 806–819.

Mode calculations for use in microwave cavity measurements of plasmas in magnetic fields.

W. P. Allis (Cambridge, Mass.)

5338:

Meynieux, Robert. Calculs approchés relatifs à la fonction Γ et à l'équation de Weber. Application à une onde électromagnétique plane qui se propage à travers une couche ionisée en présence de chocs et avec une répartition parabolique de la densité électronique en fonction de la hauteur. Ann. Télécommun. **14** (1959), 262–276.

The asymptotic behavior of the Weber function away from its turning points is investigated when the imaginary part of its index is large. The results are applied to the study of an ionized layer in an atmosphere. The electrons of the layer are assumed to have a parabolic distribution. This assumption leads to a Weber equation for the electric field. The asymptotic theory is used to calculate j, λ^{-1}, δ, $|\sigma|$, $|w_0|$, ξ, η, and $|\rho|$ in the E and F layers for experimental values of $2h$ (the thickness of the layer) and ω_m (the critical pulsation maxima).

N. D. Kazarinoff (Moscow)

5339:

Morozov, A. I.; Solov'ev, L. S. Integrals of the drift equations. Dokl. Akad. Nauk SSSR **128** (1959), 506–509 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1031–1034.

Integrals of the drift equations for a slowly varying electromagnetic field are obtained for various symmetries of the field. These integrals are then applied to the motion of particles in the field of a circular current J, on which is superposed the field of a straight wire carrying a current I. Characteristic points of the trajectories of "passed" and "blocked" particles are found under various assumptions as to the relative strengths of the fields.

I. Stakgold (Evanston, Ill.)

5340:

Kihara, Taro; Midzuno, Yukio; Kaneko, Shobu. Transport properties of plasmas in a strong magnetic field. J. Phys. Soc. Japan **15** (1960), 1101–1107.

Authors' summary: "Irreversible processes in plasmas in a strong magnetic field are discussed from both phenomenological and microscopic points of view. The thermodynamics of irreversible processes is applied and it is shown that the Onsager-Casimir reciprocity relation takes a symmetrical form for plasmas in a magnetic field. For a two-component fully ionized gas where the electrons make many free gyrations interference between electrical and thermal conduction vanish. When the mean gyration radius r_g of the electrons is shorter than the Debye length l_D, the diagonal elements of tensors of the electric conductivity and diffusion coefficient perpendicular to the magnetic field are proportional to $\ln(kTl_D/Ze^2) + \frac{1}{2}[\ln(l_D/r_g)]^2$, where Ze and $-e$ are the charges of an ion and electron respectively."

5341:

Kaji, Ikuo; Kito, Masafumi. Plasma oscillations in a magnetic field. J. Phys. Soc. Japan **15** (1960), 1851–1861.

Authors' summary: "In the presence of a uniform external magnetic field, the initial value problem for a longitudinal electron oscillation in a fully ionized plasma is treated in the range of the linear theory under the assumption that collisions are negligible and that the angle θ between wave vector and magnetic field is not equal to $\pi/2$, and $\mathrm{Re}(s) \neq 0$ on the complex s-plane. It is

shown that, if we determine the indicial condition that the solution of initial value problem varies as $\exp(i\mathbf{k}\cdot\mathbf{r}+\alpha t)$, where α is a complex valued frequency, then for $\mathrm{Re}(\alpha)<0$ (damped wave) there appears an indeterminancy in the dispersion equation of the potential, and for $\mathrm{Re}(\alpha)>0$ (amplified wave) each dispersion equation of the proper plasma oscillations, the perturbed velocity distribution function, and the potential are coincident. As a typical example, the case in which the equilibrium distribution is Maxwellian is discussed."

5342:

Majumdar, S. K. Electrodynamics of a charged particle moving through a plasma without magnetic field. Proc. Phys. Soc. **76** (1960), 657–669.

Author's summary: "The motion of a charged particle through a low density electron plasma has been investigated using equation of momentum transfer in a plasma and Maxwell's equations for electromagnetic field. It is shown that for a particle velocity greater than the average thermal velocity of the plasma electrons, a Čerenkov-like effect is set up in the plasma, only in the case of longitudinal wave motion. The existence of Mach cone is derived and the nature of the energy loss investigated."

5343:

Kovrižnyh, L. M. Effect of inelastic collisions on the velocity distribution of electrons. Ž. Èksper. Teoret. Fiz. **37** (1959), 490–500 (Russian); translated as Soviet Physics. JETP **10** (1960), 347–353.

Author's summary: "The velocity distribution function for electrons in a weakly ionized plasma has been found, taking account of inelastic collisions. It is shown that the inelastic collisions lead to a sharp drop in the distribution function for electron energies exceeding the excitation (or ionization) energy."

5344:

Ginzburg, V. L.; Gurevič, A. V. Nichtlineare Erscheinungen in einem Plasma, das sich in einem veränderlichen elecktromagnetischen Feld befindet. Fortschr. Physik **8** (1960), 97–189.

Parts 1 and 2 of this article give an excellent review of Boltzmann theory for the conductivity of an ionized gas, apparently abstracted from a number of articles by Gurevič. In particular the collision integral is treated in a novel way. Inelastic collisions as well as Coulomb interactions are treated, but not collective phenomena or the motions of ions. Run-aways are only treated in part.

Part 3 treats in detail the non-linear effects due to the heating of the electrons in the ionosphere by radio waves. This includes anomalous absorption, demodulation, cross-modulation and side-bands.

W. P. Allis (Cambridge, Mass.)

5345:

Ginsburg, V. L.; Gurevič, A. V. Nonlinear phenomena in a plasma located in an alternating electromagnetic field. Uspehi Fiz. Nauk **70** (1960), 201–246, 393–428 (Russian); translated as Soviet Physics. Uspekhi **3**, 115–146, 175–194.

See preceding review for a German version of this article.

5346:

Kuznecov, A. A. Mechanical stresses in a stationary, rotating cylinder carrying a uniform electric current. Ž. Tehn. Fiz. **30** (1960), 589–591 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 552–554.

Author's summary: "A solution is given to the problem of the mechanical stresses in a stationary, rotating cylinder carrying a uniform current with the aid of the equations of elasticity theory and with allowance for the volume electromagnetic force."

5347:

Žigulev, V. N. The phenomenon of ejection by an electrical discharge. Dokl. Akad. Nauk SSSR **130** (1960), 280–283 (Russian); translated as Soviet Physics. Dokl. **5**, 36–39.

Author's summary: "The theory of an electrical discharge in a medium of finite conductivity is examined. It is shown that an axially symmetric discharge is, in principle, accompanied by motion of the medium—the ejection effect. The laws of similarity for an axially symmetric discharge are derived, and self-similar solutions of its equations are given."

5348:

Ipatov, L. G. Propagation of electromagnetic waves in a ferromagnet. Ž. Tehn. Fiz. **30** (1960), 522–528 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 489–496.

Author's summary: "The problem of propagation of an electromagnetic wave in a poorly conducting ferromagnetic slab has been considered. It is shown that the wave propagation is effected by time-delay effects, induced currents, hysteresis and the amplitude of the wave. Taking account of these factors and using a new method for solving the differential equation

$$\frac{\partial^2 H}{\partial x^2}=(a_1+ib_1)H+(a_2+ib_2)|H|H,$$

we have been able to obtain a more general relation for propagation of the wave. The solutions which have been obtained are modified appropriately for the case of a metallic ferromagnet."

5349:

Volosov, V. I.; Čirikov, B. V. Theory of the skin effect under transient conditions. Ž. Tehn. Fiz. **30** (1960), 508–511 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 477–480.

5350:

Gordon, E. I. Charged-particle orbits in varying magnetic fields. J. Appl. Phys. **31** (1960), 1187–1190.

The author concludes: "A solution for the orbits of charged particles in varying azimuthally symmetric magnetic fields has been given in the guiding-center representation which separates the guiding center and rotational motion. It has been shown that the motion in a particular magnetic field can be determined by the solution of a first-order differential equation that determines the necessary

parameters. The invariants of the system have been discussed, and in the case of slow magnetic field variations it has been shown that the corrections to the adiabatic invariants are of the first order for a single particle. Averages over the initial conditions eliminate the first-order term so that the invariant is truly adiabatic only for a swarm of particles. When particles start with their guiding centers on the axis or with no rotational energy, the correction term for the appropriate invariant is also of the second order." *R. D. Kodis* (Providence, R.I.)

5351:

Korneenko, I. A. Mean values of the parameters in inhomogeneous media. Ž. Tehn. Fiz. **30** (1960), 44–48 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 40–44.

The author derives average or effective electrical parameters for an inhomogeneous medium by using Green's theorem to connect the interior fields with the boundary fields of the various internal components of the medium. Two examples are given: the case of a plane laminar medium and the case of a medium with spherical inhomogeneities. The method apparently works only when the average effect is isotropic. *I. Kay* (New York)

5352:

Poritsky, H. Helical fields. J. Appl. Phys. **30** (1959), 1828–1837.

Author's summary: "This paper is devoted to a study of helical fields, that is, fields which are invariant under screw motions of space which move a certain helix into itself.

"Simple, analytic, helically invariant solutions of the Laplace equation are given to describe the electrostatic field of a charged helix and the magnetic field of a helical electric current. A flux function ψ is introduced for solenoidal helical vector fields and differential equations are derived for the potential function ϕ and the flux functions ψ. Certain graphical flux plotting methods are outlined and illustrated and network analogies are suggested for solving these fields."

A. J. Estin (Boulder, Colo.)

5353:

Minkov, I. M. Electrostatic field of a condenser with dielectric insert. Ž. Tehn. Fiz. **30** (1960), 1207–1209 (Russian); translated as Soviet Physics. Tech. Phys. **5** (1961), 1143–1146.

5354:

Lindsay, P. A. Application of the relaxation method to the solution of space charge problems. J. Electronics Control (1) **6** (1959), 415–431.

The author describes a numerical method of solution of the space charge equation in one dimension. The two problems treated explicitly are: space charge between two concentric circular cylinders and between two concentric spheres, the problems being of rotational and spherical symmetry respectively. The differential equations are converted into finite difference approximations. Starting with a linear potential distribution as a first approximation the subsequent approximations follow from the relaxation procedure. Several typical cases, as to boundary values, are considered. Convergence is rapid at low space charge densities and poorer at high densities, as is shown by numerical values, contained in several tables. Initial electron velocity must be single-valued. The method breaks down if this is not satisfied.

M. J. O. Strutt (Zürich)

5355:

Konstantinov, O. V.; Perel', V. I. Possible transmission of electromagnetic waves through a metal in a strong magnetic field. Ž. Èksper. Teoret. Fiz. **38** (1960), 161–164 (Russian. English summary); translated as Soviet Physics. JETP **11**, 117–119.

Authors' summary: "It is shown that an electromagnetic wave propagating along a magnetic field can penetrate a metal plate perpendicular to the field if the Larmor frequency is higher than the frequency of the propagating wave and much higher than the collision frequency, and if the electron Larmor radius is smaller than the wavelength in the metal."

5356:

Vinti, John P. ★Theory of the multipath propagation of frequency modulated waves. Ballistic Research Laboratories Report No. 1025, 1957. Aberdeen Proving Ground, Md. Distributed by Office of Technical Services, U.S. Dept. Commerce as PB 151117. 254 pp. $4.00.

The theory is developed for the combination of a direct ray and one reflected ray. Under the assumption that transmitting and receiving antennae are broadband (constant modulus, linear phaseshift) and that neither ray is too near a frequency-dependent null of an antenna, the voltage produced in the receiving antenna is calculated: $V=F(\cos b_1+R\cos b_2)$. The carrier wave is modulated with a multitone signal, all tones harmonics of one frequency. The author calculates the phase-deviations in the received demodulated tones (for the measurement of distance). For the special case $R=1-0$ (equal amplitudes of direct and reflected ray) the integrals for the phase-errors can be evaluated in closed form. [Cf. Vinti and Leser, J. Soc. Indust. Appl. Math. **5** (1957), 15–31; MR **19**, 850.] For other cases Maclaurin or Fourier series are used to derive numerical results.

F. L. H. M. Stumpers (Eindhoven)

5357:

Einspruch, Norman G.; Truell, Rohn. Propagation of traveling waves in a circular cylinder having hexagonal elastic symmetry. J. Acoust. Soc. Amer. **31** (1959), 691–693.

Authors' summary: "The problem of propagation of traveling waves in a right circular cylinder having hexagonal elastic symmetry is considered here. Exact expressions for the displacements produced by both the compressional and torsional modes are obtained. A conclusion of importance in ultrasonic methods is the result that the compressional and torsional modes propagate independently without coupling either through the equations of motion or the boundary conditions. A condition relating the frequency and wave number of the torsional wave is derived."

5358:

Mahan, A. I.; Bone, L. P. Far-field diffraction properties of a plane-parallel plate when placed partially in front of a rectangular diffracting aperture. J. Opt. Soc. Amer. **50** (1960), 683–697.

5359:

Ter-Mikaelyan, M. L. A study of the limits of applicability of the theory of ionization losses. Ž. Èksper. Teoret. Fiz. **38** (1960), 882–888 (Russian. English summary); translated as Soviet Physics. JETP **11**, 637–641.

Author's summary: "The energy losses of an arbitrary moving particle are calculated by means of the macroscopic Maxwell equations. A separation into ionization losses and radiation losses is made, outside the framework of perturbation theory. Effects on the formulas for ionization losses owing to multiple Coulomb scattering are examined, and also effects of the finiteness of the path length. It is found that because of the existence of the density effect the influence of multiple Coulomb scattering on this part of the losses can be neglected."

5360:

Bugnolo, Dimitri S. Transport equation for the spectral density of a multiple-scattered electromagnetic field. J. Appl. Phys. **31** (1960), 1176–1182.

This paper discusses the propagation of electromagnetic fields in a medium whose index of refraction is a random function of the coordinates and of time. A transport equation is derived for the spectral density, which is more general than the Born approximation (first-order perturbation theory), whose validity is limited to problems in which the mean free path of a photon is large compared to the length of the optical path. The general theory is applied to some cases which possess practical interest, particularly propagation in the troposphere.

P. G. Bergmann (Syracuse, N.Y.)

5361:

Kay, Irvin. The inverse scattering problem when the reflection coefficient is a rational function. Comm. Pure Appl. Math. **13** (1960), 371–393.

Under proper restrictions on $V(x)$, the solutions $u(x, k)$ of $d^2u/dx^2+(k^2-V)u=0$ $(-\infty<x<\infty)$ behave at infinity according to $u(x, k)\sim A_\pm(k)\exp(ikx)+B_\pm(k)\exp(-ikx)$ $(x\to\pm\infty)$. In terms of the coefficients A and B, the scattering matrix S is defined by

$$\begin{pmatrix}A_+\\ B_-\end{pmatrix}=S\begin{pmatrix}A_-\\ B_+\end{pmatrix},\quad S=\begin{pmatrix}t(k) & \rho(k)\\ r(k) & \tau(k)\end{pmatrix}.$$

The author determines V, the remaining elements of S, and two linearly independent solutions u if the reflection coefficient $r(k)$ is a properly restricted rational function of k.

C. J. Bouwkamp (Eindhoven)

5362:

Papadopoulos, V. M. Diffraction of a pulse by a resistive half-plane. I. Normal incidence. Proc. Roy. Soc. London. Ser. A **255** (1960), 538–549.

This paper deals with the diffraction of a plane pressure pulse with Heaviside unit function time dependence, by a semi-infinite screen, subject to the boundary condition that the transverse velocity of the screen is at every point the same constant multiple of the pressure jump across the screen. Since there is no characteristic length, the solution depends only on r/ct and θ, where (r, θ) are polar coordinates, the screen being $\theta=0$ or $\theta=2\pi$, t is the time and c the velocity of sound. As in the analogous case of 'cone-fields' in linearized supersonic flow theory, the problem can be reduced to that of the determination of an analytic function regular in a half-plane whose real and imaginary parts are related in a known way on the real axis, and which is subject to conditions on the permissible singularities. It is found that outside the diffraction front $r=ct$, the field is similar to that in the steady problem of reflexion and transmission by a resistive surface. Inside the diffraction front the field is modified by a diffracted wave, and one notable feature is that the pressure and radial velocity on the screen remain constant for $ct>r$.

The author also considers the analogous electromagnetic problem of diffraction by a screen with finite constant surface resistance. This is mathematically identical with the acoustic problem when the electric field is radial and the magnetic field is parallel to the edge of the screen. The other case, where the electric field is parallel to the edge of the screen, is different, but can be reduced to the problem already solved after further analysis.

F. G. Friedlander (Cambridge, England)

5363:

Papadopoulos, V. M. Diffraction of a pulse by a resistive half-plane. II. Oblique incidence. Proc. Roy. Soc. London. Ser. A **255** (1960), 550–557.

A solution of the three-dimensional scalar wave equation with wave velocity c that depends on a space coordinate z and the time t in such a manner that it is a function of $\tau=t-x/lc$, where $l>1$, satisfies a two-dimensional wave equation with respect to the other two space variables and τ, with wave velocity $lc/(l^2-1)^{1/2}$. This principle can be used to deduce the solution of a diffraction problem with an obliquely incident plane wave from the two-dimensional case of normal incidence. It is used here to generalize the results of the previous paper. In the electromagnetic case, however, there is a difficulty since there is interaction between pulses of different polarizations. This is overcome by splitting a general plane pulse into two independent oblique electric and magnetic polarizations that can be treated separately.

F. G. Friedlander (Cambridge, England)

5364:

Mertens, Robert. The diffraction of light by two superposed parallel supersonic waves being harmonics of the same fundamental. Proc. Indian Acad. Sci. Sect. A **50** (1959), 289–302.

The author previously considered the title problem on the assumption that the ratio of the frequencies of the two supersonic waves was an integer [same Proc. **48** (1958), 288–304; MR **20** #7498]. He now extends his analysis to frequency ratios of incommensurable integers.

J. W. Miles (Los Angeles, Calif.)

5365:

Zitron, N. R. Shielding of transient electromagnetic signals by a thin conducting sheet. J. Res. Nat. Bur. Standards Sect. D **64D** (1960), 563–567.

Author's summary: "The shielding effect of a thin, horizontal imperfectly conducting sheet against the transient field of a vertical magnetic dipole when excited by a ramp function is investigated. The results are calculated by taking Laplace transforms of the frequency spectrum functions for the steady-state problem. The response to the ramp function is calculated and the significance of the results in shielding against surges is discussed."

5366:

Piefke, G. A contribution to the theory of corrugated guides. J. Res. Nat. Bur. Standards Sect. D **64D** (1960), 533–555.

Author's summary: "The transmission characteristics of certain structures belonging to the class of corrugated guides are calculated by means of a new method. It is assumed that the guide wavelength always is much greater than the corrugation constant. The periodical structure of the guide is therefore replaced by a quasi-homogeneous, but anisotropic medium.

"The following structures are studied: The 'ring-element guide', which consists of an axial stack of insulated metallic rings with arbitrary surrounding medium; the 'disk guide', which is a ring-element guide with infinite radial extension of the rings; the 'disk loaded waveguide', and the 'corrugated waveguide'.

"As a rule guides can propagate modes with a phase velocity $v_p > c$ (c = velocity of light) and modes with $v_p < c$. The capability of the various modes depends on the losses of the guide. The ring-element guide is well suited for transmission with the H_{01}-mode since, except the H_{0n}-modes, all modes may be highly attenuated (mode filters). As delay lines ($v_p < c$), all guides have band pass characteristics."

5367:

Marini, John W. Radiation and admittance of an insulated slotted-sphere antenna surrounded by a strongly ionized plasma sheath. J. Res. Nat. Bur. Standards Sect. D **64D** (1960), 525–532.

Author's summary: "Given the voltage distribution along the slot, expressions for the radiation pattern, input admittance, and the external efficiency of an insulated slotted-sphere antenna surrounded by a homogeneous, isotropic, strongly ionized sheath are obtained.

"At low frequencies the input impedance is proportional to the sum of the intrinsic impedance of the sheath and an equivalent inductance due to the insulating coating, the radiation pattern reduces to that of a small loop, while the external efficiency is the product of three factors arising because of the power dissipated in the sheath by higher order modes that contribute little to the radiation field, attenuation through the sheath of the modes that do radiate, and reflection loss of these modes at the outer surface of the sheath.

"Since the reflection loss decreases with increasing frequency while the attenuation increases, there exists an optimum frequency of operation. At this frequency, the ionized sheath has a thickness equal to two-and-one-half skin depths."

5368:

Kuznecov, A. A. Mechanical stresses produced by the radial electromagnetic force in a multilayer coil wound with wire of rectangular cross section carrying a uniform current. Ž. Tehn. Fiz. **30** (1960), 592–597 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 555–561.

Author's summary: "The solution is given to the problem of the mechanical stresses in a multilayer coil by means of the equations from elasticity theory and with allowance for the electromagnetic force."

5369:

Zemanian, Armen H. Further properties of certain classes of transfer functions. Quart. Appl. Math. **18** (1960/61), 223–228.

The Laplace transform of the impulsive response of a linear system is termed the system transfer function. Some properties of certain defined classes of the transfer function, which need not be rational, are adduced, and bounds on certain derivatives of the corresponding impulsive response functions are obtained.

R. Kahal (Washington, D.C.)

CLASSICAL THERMODYNAMICS, HEAT TRANSFER

See also 5268, 5271, 5330.

5370:

Birkhoff, G.; Margulies, R. S.; Horning, W. A. Spherical bubble growth. Phys. Fluids **1** (1958), 201–204.

The rate of growth of vapor bubbles in a superheated liquid and of spherical gas bubbles in liquid supersaturated with gas is investigated. An asymptotic expression for the bubble radius due to Plesset and Zwick is obtained without making their hypothesis of a localized temperature drop at the wall of the bubble. The mathematical analysis of the problem involves similarity assumptions and leads to an ordinary differential equation whose explicit solutions must be examined.

P. R. Garabedian (New York)

5371:

Rudin, Morton. Criteria for thermodynamic equilibrium in gas flow. Phys. Fluids **1** (1958), 384–392.

5372:

Murgai, M. P.; Emmons, H. W. Natural convection above fires. J. Fluid Mech. **8** (1960), 611–624.

The natural convection above fires in a calm dry atmosphere is treated on the assumption that the convection column is wholly turbulent. By defining suitable dimensionless mean quantities and assuming that a "shape parameter" for the column is constant with altitude, the problem is reduced to a set of ordinary differential equations. These have been solved on a Pace computer for a range of (constant) values of atmospheric lapse rates and initial plume velocities, and the solution curves are presented in the paper.

The convection column can then be calculated for any atmospheric conditions by dividing the atmosphere into regions over which the lapse rate is constant. A sample calculation is given.

J. F. Clarke (Cranfield)

5373:

Tirskiĭ, G. A. The heating of a heat-conducting wall behind a moving compression shock. Dokl. Akad. Nauk SSSR **128** (1959), 1140–1143 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 981–984.

A system of differential equations, describing the heat flow in a semi-infinite half-space impinged by a shock-wave, is transformed to the integral equation form. A method for solving this system numerically is proposed.

H. Polachek (Silver Spring, Md.)

5374:

Vodička, Václav. Steady temperature in an infinite multilayer plate. Z. Angew. Math. Mech. **40** (1960), 161–165.

Let $u_i(x, y)$ satisfy Laplace's partial differential equation in the strips $0 \leq y_i < y < y_{i+1}$, $-\infty < x < \infty$ $(i = 1, 2, \cdots, n)$. At the interfaces $y = y_{i+1}$ $(i = 1, 2, \cdots, n-1)$ those functions are to satisfy the conditions

$$\partial u_i/\partial y = h_{i+1}(u_{i+1} - u_i), \quad \partial u_{i+1}/\partial y = h'_{i+1}(u_{i+1} - u_i),$$

where the constants h_i and h_i' are positive. The outer boundary conditions are $u_1(x, y_1) = f(x)$ and $u_n(x, y_{n+1}) = F(x)$, $-\infty < x < \infty$, where f and F are given periodic functions of period 2π, represented by their Fourier series. The unknown functions u_i are also assumed to be periodic in x: $u_i(x + 2\pi, y) = u_i(x, y)$. The author solves this boundary value problem by representing $u_i(x, y)$ in series whose coefficients are given explicitly, using superposition processes.

R. V. Churchill (Ann Arbor, Mich.)

5375:

Vodička, Václav. Stationary temperature fields in multilayer cylindrical tubes. Z. Angew. Math. Mech. **40** (1960), 165–170. (German and Russian summaries)

Using his formula derived in the paper reviewed above, for the steady temperatures in a composite slab, the author solves the corresponding boundary value problem for the laminated circular cylinder. Here $u_i(\rho, \theta)$ satisfies Laplace's equation in polar coordinates in the annulus $\rho_{i+1} < \rho < \rho_i$, $0 \leq \theta \leq 2\pi$ $(i = 1, 2, \cdots, n)$ and the conditions $u_1(\rho_1, \theta) = f(\theta)$, $u_n(\rho_{n+1}, \theta) = F(\theta)$, and $\partial u_i/\partial \rho = h_{i+1}(u_i - u_{i+1})$, $\partial u_{i+1}/\partial \rho = h'_{i+1}(u_i - u_{i+1})$ when $\rho = \rho_{i+1}$ $(i = 1, 2, \cdots, n-1)$. Also, an approximate solution of the problem for the laminated elliptical cylinder is written for the case in which the elliptical interfaces, all confocal with the outer ellipses, are nearly circular.

R. V. Churchill (Ann Arbor, Mich).

5376:

Menkes, J. On the stability of a plane deflagration wave. Proc. Roy. Soc. London. Ser. A **253** (1959), 380–389.

The undisturbed temperature is assumed representable by $\tanh \alpha x$, α a constant, $|x| < \infty$. Unit Lewis number is considered, so the concentration can be determined from the temperature. If the enthalpy perturbation is written as $p^*(x) \exp(-ct')$, where t' is the time and c a constant, the linearized inviscid stability problem reduces to a second order ordinary differential equation for p^*, with $p^*(\pm\infty) = 0$. The resultant condition on the eigenvalue $c \equiv k + i\omega$ is $\text{Re}\sqrt{(1-4c)} \geq 1$. The case $\omega = k = 0$ is discussed separately. For every $\omega \neq 0$, one can choose $k > 0$; the author states that this means stability. His statement would seem to require justification, as negative values of k are also possible for a given non-zero ω.

L. A. Segel (Troy, N.Y.)

5377:

Kröger, F. A.; Stieltjes, F. H.; Vink, H. J. Thermodynamics and formulation of reactions involving imperfections in solids. Philips Res. Rep. **14** (1959), 557–601. (French and German summaries)

QUANTUM MECHANICS

See also A4811, 5172, 5442, 5448, 5450.

5378:

Datzeff, A. B. Sur l'interprétation de la mécanique quantique. II. Détermination de la probabilité de présence. J. Phys. Radium **21** (1960), 201–211. (English summary)

Author's summary: "Taking into consideration his previous work [same J. **20** (1959), 949–955; MR **22** #2346] the author formulates the exact initial conditions to which is subjected the probability density of occurrence of positions $w(x, y, z)$ of a microcorpuscule μ. It is shown that the function $f(x, y, z)$, in which $|f|^2 = w$, should satisfy a differential equation of the Sturm-Liouville type. The initial conditions determine its coefficients in a unique way, thus getting a probability equation for f, identical to Schrödinger's equation for this case. The following cases are summarized: a problem with two or three dimensions, in case of presence of electromagnetic potentials, in case of dependence of time, and in case of numerous corpuscules.

"In all those cases one comes to the corresponding Schrödinger equation."

5379:

Grigor'ev, V. I.; Myakišev, G. Ya. Virtual and real transitions in quantum theory. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1958**, no. 6, 71–75. (Russian)

Applications of time-dependent perturbation theory, in second and higher orders, are usually expressed in terms of the concept of "virtual" states of quantum mechanical systems. Much consideration has been given in the physical literature to the distinction between virtual states and the real, physical, states which occur in actual fact. The authors give a new discussion of this question from the point of view of perturbation theory as a solution of an initial-value problem. They show that the solution of a perturbation problem does not depend in an essential manner on the separation into virtual and real states. {Since no use is made of the concept of a complete Hilbert space of states, the discussion given does not resolve the more basic problems concerning virtual states as a basis for such a space.}

E. L. Hill (Minneapolis, Minn.)

5380:

Furry, W. H.; Ramsey, N. F. Significance of potentials in quantum theory. Phys. Rev. (2) **118** (1960), 623–626.

Recently, Aharonov and Bohm [Phys. Rev. (2) **115**

(1959), 485–491; MR 22 #1336] demonstrated by two simple thought experiments that electromagnetic potentials could be expected to have experimental effects in quantum mechanics. These rather surprising conclusions followed from an analysis of a split beam type of experiment in which the separate beams were subject to different potentials (either scalar or vector) although the field strengths were at all times zero in the neighbourhood of the beam. On recombining the two portions of the beam, interference fringes would be observed arising from the unequal phase changes suffered by the two wave packets representing the separate portions.

In this paper, it is pointed out that this effect would apparently entail the possibility of determining whether a particle was in one beam or the other without destroying the interference pattern. This would be in contradiction with the principles of quantum mechanics. An analysis of the two experimental arrangements of Bohm and Aharonov is made, when an additional detection is present to locate which beam contains the particle. It is shown that when sufficient precision in the position measurement is achieved, then a compensating uncertainty in the scalar or vector potential appears which is just sufficient to destroy the interference fringes. *C. A. Hurst* (Adelaide)

5381:

Sato, Shigeo. Second quantization and Lorentz invariance. Progr. Theoret. Phys. **23** (1960), 717–730.

In this paper Lorentz invariant theory of second quantization in terms of particle representation with creation-annihilation operators is discussed. It is pointed out that Hamiltonian formalism leads to unsatisfactory results and hence the theory of Yang-Feldman's S matrix is considered together with applications to Compton scattering and decay interactions. The results are obtained by extending Foldy's formalism [L. L. Foldy, Phys. Rev. (2) **102** (1956), 568–581; MR **20** #709] to the present case. It is shown that Maxwell's equations can be reduced to the canonical form. Apart from this last, the theory must be regarded as a first attempt at including second quantization in Foldy's synthesis. The results do not suggest any answer to the questions raised by Foldy; in particular, whether the change in charge conjugation parity on mutual annihilation of a particle and its antiparticle in a given state can be represented. It is difficult to see, also, that the nonexistence of solution of equation (3.11) on which part of the criticism of Hamiltonian theory is based is demonstrated. It is such a theory that forms the guiding principle of Wigner's program [E. P. Wigner, Nuovo Cimento (10) **3** (1956), 517–538; MR **18**, 173]. It must be remarked also that the S matrix can be determined from the functional equation $i(\delta S(\sigma)/\delta\sigma(x)) = H'(x:n)S(\sigma)$ on a space-like surafce σ, which defines the interaction Hamiltonian H' [H. Umezawa, *Quantum field theory*, Interscience, New York, 1956; MR **20** #690]. *A. H. Klotz* (Newcastle-upon-Tyne)

5382:

Martin, A. Analytic properties of $l\neq 0$ partial wave amplitudes for a given class of potentials. Nuovo Cimento (10) **15** (1960), 99–109. (Italian summary)

The scattering of waves of non-zero angular momentum is studied for the family of potentials

$$\exp(-\mu r)\int_0^\infty C(\alpha)\exp(-\alpha n)\,d\alpha.$$

The scattering amplitude is analytic for $\mathrm{Im}(k)>0$ except for poles due to the bound states on the imaginary axis and a cut from $i\mu/2$ to $i\infty$. A dispersion relation can be derived. *C. Strachan* (Aberdeen)

5383:

Kaschluhn, F. Eine feldtheoretische Verallgemeinerung der Impulsnäherung. Nuclear Phys. **14** (1959), 314–338. (English summary)

The impulse approximation is discussed in the context of pion-deuteron scattering as described by the non-relativistic pseudo-vector interaction of extended source nucleons with second quantized relativistic mesons. The S-matrix is defined in terms of bare deuteron (or bare pion-deuteron) states which are chosen to be the projection of the physical (pion-) deuteron state on the meson vacuum (or on one meson state). The interaction Hamiltonian is separated into a part responsible for energy shifts and the potential which binds the deuteron, and a part which causes pion-deuteron scattering but does not lead to energy shifts. Only this latter part is switched off adiabatically as $t\to\pm\infty$ so that physical states go into bare states but the deuteron remains bound. The impulse approximation is exhibited explicitly. The final results are a formula for the T-matrix for pion-deuteron scattering in terms of the T-matrix elements for pion-single nucleon scattering and the deuteron form factor, and a similar formula for the pion-deuteron vertex function.

O. W. Greenberg (Cambridge, Mass.)

5384:

Oehme, Reinhard. Some analytic properties of the vertex function. Phys. Rev. (2) **117** (1960), 1151–1159.

In this paper the author determines the domain D of two complex invariants p_1^2, p_2^2 for which the vertex function $F(p_1^2, p_2^2, (p_1+p_2)^2)$ is analytic in the cut plane of the third variable $(p_1+p_2)^2$. The properties used are causality, spectral conditions and Lorentz invariance. It is shown that D cannot be extended by further use of these properties due to the fact that examples based on the lowest order perturbation diagram may be constructed which satisfy the properties and have singularities at each point of the boundary of D. However certain of these examples arise from perturbation diagrams in which nucleon conservation is violated, so that correct use of further properties may enlarge D. When singularities may appear in the $(p_1+p_2)^2$ plane it is shown that, when p_1^2 and p_2^2 are real and below their static cuts these singularities are confined to a finite region surrounding the low energy part of the static cut in the $(p_1+p_2)^2$ plane. An upper bound to the size of this region is obtained.

John G. Taylor (Paris)

5385:

Zel'dovič, Ya. B. Scattering by a singular potential in perturbation theory and in the momentum representation. Ž. Èksper. Teoret. Fiz. **38** (1960), 819–824 (Russian. English summary); translated as Soviet Physics. JETP **11**, 594–597.

Author's summary: "A method is developed for treatment of scattering by a singular potential in the

momentum representation and in perturbation theory. Application of such renormalization techniques permits one to derive well-known results for cross sections, despite the fact that the integrals diverge and the matrix elements entering into the wave equation in the momentum representation vanish."

5386:

Ryazanov, M. I. On the processes involving transfer of momentum to the medium. Ž. Èksper. Teoret. Fiz. **38** (1960), 854–862 (Russian. English summary); translated as Soviet Physics. JETP **11**, 617–623.

Author's summary: "The change of the transition probability caused by the Coulomb scattering of particles by the atoms of the medium is found for a certain class of processes in which there are one charged particle and an arbitrary number of neutral particles in the initial and final states."

5387:

Chang, T. S. Remarks on Chew-Low equations. Sci. Sinica **9** (1960), 466–474.

By using the definitions $H_0\psi_0=0$, $H\Psi_0=0$, $H=H_0+V$, the Chew-Low equation is transformed into

$$\Psi_q = A_q{}^+\psi_0' - \lim_{E\to 0}\frac{1}{i(H_0-\omega_q)+E}\,iV\Psi_q,$$

the usual scattering equation except that $\psi_0'=\langle\psi_0, \Psi_0\rangle\psi_0$ replaces ψ_0. Similar comparisons for $\psi_{q_1q_2}\cdots$ are made. Since $(\psi, \Psi_0)\neq 1$ the wave-functions in the Chew-Low theory and those of the conventional theory cannot both be normalized. The same result is obtained working from formal scattering theory. The author is rederiving the result that if ψ_0 (the bare nucleon wave-function) is normalized, then the physical nucleon state

$$\Psi_0 = \lim_{t\to\infty} e^{iHt}e^{-iH_0t}\psi_0 = \lim_{\varepsilon\to 0}\frac{\varepsilon}{iH-\varepsilon}\,\psi_0$$

requires renormalization.

In the second part of the paper the author discusses the existence of solutions of an integral equation of the Chew-Low type, as done by K. W. Ford [Phys. Rev. (2) **105** (1957), 320–327; MR **18**, 626]. The method is also applied to equations satisfying crossing symmetry, and it is shown that no solutions exist unless ghost states occur.

R. F. Streater (Princeton, N.J.)

5388:

Tani, Smio. Formal theory of scattering in the quantum field theory. Phys. Rev. (2) **115** (1959), 711–720.

The transformation operator U from the eigenfunctions of the unperturbed to the perturbed Hamiltonian is expressed in exponential form and a representation is defined in which "all effects of the self-field of physical particles on themselves are eliminated while the remainder is presented in the form of an effective velocity-dependent potential among them". The possibility of an approximate calculation of U with the help of the Born approximations for the phase shifts is discussed and it is pointed out that the present approach has the advantage over that of Low, for example, that approximations may be introduced without destroying the unitarity of the S matrix. A disadvantage, compared with Low's theory of scattering, is the difficulty of treating renormalization. The method is also adapted to using scattering data and the first and second Born approximations to partially determine the effective potential functions.

A. J. Coleman (Kingston, Ont.)

5389:

Sokolov, A. A.; Kolesnikova, M. M. The behavior of fermion spin in elastic scattering. Ž. Èksper. Teoret. Fiz. **38** (1960), 165–171 (Russian. English summary); translated as Soviet Physics. JETP **11**, 120–124.

Authors' summary: "The behavior of spin in the elastic scattering of longitudinally polarized fermions and its dependence on the character of the interaction are investigated. It is shown that in the ultrarelativistic case (or for fermions with zero rest mass) the angle between the spin and the momentum is unchanged in V and A interactions, but the spin flips in S, P, and T interactions."

5390:

Taylor, John G. Limitations to dispersion relations. Ann. Physics **10** (1960), 516–535.

Proofs of dispersion relations for scattering processes expressing the analyticity of the amplitudes as a function of energy for fixed momentum transfer have been given for many processes but always with an upper limit on the momentum transfer. By a careful study of an example in perturbation theory the author shows that a certain singularity in the absorptive part of the amplitude cancels, which cancellation cannot be seen in the general (non-perturbative) derivations. He further claims that under circumstances where anomalous thresholds must be expected the familiar methods of derivation by a continuation in the projectile mass fails even in fourth order perturbation theory. Actually recent developments in the anomalous threshold problem enable one to overcome this difficulty.

M. L. Goldberger (Princeton, N.J.)

5391:

Ritus, V. I. The relativistically covariant spin structure of the S matrix. Ž. Èksper. Teoret. Fiz. **38** (1960), 1489–1498 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1075–1081.

This paper is concerned with exhibiting the spin structure of the S-matrix in a relativistically covariant way for a variety of processes involving particles and photons. Most of the attention is directed towards particles (and anti-particles) of spin zero and one-half although some of the results apply to spin one. The object of the paper is to relate the scattering matrix in the center of mass frame to that in a general frame. Most of the results are well known.

M. L. Goldberger (Princeton, N.J.)

5392:

Balachandran, A. P.; Ranganathan, N. R. A note on scattering and production amplitudes. Nuclear Phys. **18** (1960), 81–84.

Sucher and Day have obtained a formal expression for the scattering amplitude of the process $A+B\to C+D$ in terms of the scattering amplitudes of $A+B\to A+B$ and $C+D\to C+D$, and the "pure production" amplitude in

which intermediate states with either the initial particles or the final particles are excluded.

Following their methods the present authors obtain a formal relation between scattering and production amplitude when arbitrary numbers of channels are open. Possible application of this equation to a few problems is suggested. The question of showing the utility of this relation by performing the explicit calculation along this line applied to any problem is left untouched.

T. Sasakawa (Cambridge, Mass.)

5393:

Siokos, Theodore Chr. Quantic theory and field of gravity. Prakt. Akad. Athēnōn **33** (1958), 17–23 (1959). (Greek. English summary)

5394:

Fried, Herbert M. Example of a soluble field theory with finite charge renormalization. Phys. Rev. (2) **118** (1960), 1427–1429.

This paper discusses a model which could perhaps be called a "completely non-relativistic Lee model". In the conventional Lee model one assumes that there is the usual relativistic relation between the energy and the momentum of the light particles, viz., $\omega=\sqrt{(\bar{k}^2+\mu^2)}$. The author replaces this formula by the non-relativistic formula $\omega=\mu+\bar{k}^2/2\mu$. This improves the convergence of some of the integrals in the model and, in particular, makes the renormalization of the coupling constant a finite number. Roughly speaking, one can say that this model is equivalent to the conventional Lee model with a particular cut-off function. Consequently, one cannot expect any drastic changes in the basic features of the model as a consequence of the non-relativistic approximation alone. This expectation is also borne out by the author's computations which show the appearance of anomalous states (ghosts) in the usual way.

G. Källén (Lund)

5395:

Jahn, H. A.; Howell, K. M. New (Regge) symmetry relations for the Wigner 6j-symbol. Proc. Cambridge Philos. Soc. **55** (1959), 338–340.

Starting with a generating function of Schwinger with integral parameters, the authors derive new symmetry relations for Wigner 6-j symbols (or Racah coefficients) employed by G. Racah [Phys. Rev. (2) **62** (1942), 438–462], in the theory of electron interactions. New notation enables them to prove the proportionality of the coefficients to a generalised hypergeometric function (Saalschützian ${}_4F_3$) of unit argument, associating distinct values of the symbols to ordered partitions of a specified integral parameter, one of seven used. This results in a considerable simplification in computing tables of the coefficients. The same symmetries have been pointed out, independently, by T. Regge [Nuovo Cimento (10) **11** (1959), 116–117], as the authors note, whose notations resemble theirs.

A. H. Klotz (Newcastle-upon-Tyne)

5396:

Freund, P. G. O. About the concept of particle in quantum field theory. Nuovo Cimento (10) **14** (1959), 673–680. (Italian summary)

It is suggested that the divergencies in field theory are due to the infinite fluctuations pertaining to the field operators on account of their not commuting with the particle number operators for all of which the vacuum is an eigenstate. States with defined numbers of particles should be made unphysical by non-commutation of number operators for states of different particles. For the Hamiltonian this leads to coupling of different oscillators and a cut-off in the Green's function integral. The concept of particle loses its physical meaning.

C. Strachan (Aberdeen)

5397:

Stépanov, Boris. Variables dynamiques et intégrales du mouvement en théorie des champs quantifiés. Cahiers de Phys. **13** (1959), 173–190.

This paper deals mainly with some extensions of the ideas of Bogolubov and Shirkov [*Introduction to the theory of quantized fields*, Interscience, New York, 1959; MR **22** #1349; Chap. III] where a classical function $g(x)$ was introduced into the S-matrix to discuss the adiabatic decoupling. The author considers functions such that $g(x)$ is asymptotically zero in the far future and unity in the far past. Constants of the motion such as the total field momentum are then written as $\int T^{\mu\nu}\partial_\nu g(x)\,d^4x$ ($T^{\mu\nu}$ the stress tensor). This expression is independent of the form of $g(x)$, and for the special choice of a Heaviside step function, $\theta(x^0)$, reduces to the usual result. The function $g(x)$ is introduced into the interaction Lagrangian and various S-matrix relations obtained in terms of variational derivatives of S with respect to $g(x)$. A generalized Tomanaga-Schwinger state vector equation is derived which reduces to the usual one for the special choice $g=\theta$. Generalized expressions for "annihilation" and "creation" operators in the presence of interactions are also obtained. The relation between the author's approach and the standard field theory Green's functions are discussed.

R. Arnowitt (Syracuse, N.Y.)

5398:

Gupta, V. Strong interactions and isobaric gauge invariance. Nuclear Phys. **18** (1960), 149–152.

5399:

Ansel'm, A. A. On certain general properties of the photon propagation function in quantum electrodynamics. Ž. Èksper. Teoret. Fiz. **38** (1960), 1288–1296 (Russian. English summary); translated as Soviet Physics. JETP **11**, 929–935.

Author's summary: "By considering jointly the spectral representation of the photon Green's function and the renormalizability property, the behavior of the D function is investigated for very large energies and $e^2=1/137$, and for very large charges but not too high energies. With an accuracy to within a numerical parameter it was possible to establish the dependence of the D function on charge in the first case, and on energy in the second case."

5400:

Lipmanov, È. M. On the analogy between the weak and the electromagnetic interactions. Ž. Èksper. Teoret. Fiz. **38** (1960), 1233–1236 (Russian. English summary); translated as Soviet Physics. JETP **11**, 891–893.

Author's summary: "The analogy between the weak and the electromagnetic interactions is presented in such a way that the electric current and the charged currents in the weak interaction are obtained from a single symmetrical expression which involves the operators $\frac{1}{2}+\tau$ and $1+\gamma_5$ after the requirements of conservation of the electric, leptonic, and baryonic charges and of vanishing of the photon mass are imposed. A definite 'chirality' is ascribed to particles of half-integral spin, which is conserved in weak interactions. Doublets of 'bare' Fermi particles in the weak and electromagnetic interactions are classified in terms of the values of the electric charge, the leptonic or baryonic charge, and the chirality."

5401:

Ioffe, B. L. Renormalization in parity-nonconservation theory. Ž. Èksper. Teoret. Fiz. **38** (1960), 1263–1275 (Russian. English summary); translated as Soviet Physics. JETP **11**, 911–919.

Author's summary: "A method is proposed for the renormalization of mass, charge, and wave functions in the parity-nonconservation theory. The method is checked in the case in which the 'three-Γ approximation' equation is used for the vertex part."

5402:

Sasaki, Seibun. Remarks on the gauge transformations. Kumamoto J. Sci. Ser. A **4**, 96–98 (1959).

5403:

Łopuszański, Jan. The Ruijgrok-Van Hove model of field theory in terms of "dressed" operators. Physica **25** (1959), 745–764; erratum, 1368.

The author studies the field theory model due to Ruijgrok and Van Hove [Physica **22** (1956), 880–886; MR **18**, 626]. He finds the similarity transformation which transforms the "bare" creation operators of the model into the minimal "dressed" creation operators which create the asymptotically stationary states of Van Hove [ibid. **21** (1955), 901–923; **22** (1956), 343–354; MR **21** #3218, 3219]. He expresses the Hamiltonian of this model in terms of these dressed operators and uses this formulation to discuss in detail the dependence of the theory on the cut-off parameter, the eigenvalue problem for states of one nucleon and one meson, and the occurrence of an indefinite metric in the "Hilbert" space of states for values of the cut-off parameter beyond the critical value, among other things. The author corrects Eq. (23), p. 752, in the erratum and points out that his solution of the one-nucleon—one-meson problem is not exact, but is a first order Tamm-Dancoff approximation.

O. W. Greenberg (Cambridge, Mass.)

5404:

Łopuszański, J. The Ruijgrok-Van Hove model expressed in terms of "dressed" operators. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. **7** (1959), 441–447. (Russian summary, unbound insert)

This paper is an earlier and less detailed version of the paper by the same author reviewed above.

O. W. Greenberg (Cambridge, Mass.)

5405:

Kogan, R. M. On the statistics of radioactive transformations of atomic nuclei forming a family. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1960**, 680–687. (Russian)

5406:

Asaad, W. N. Relativistic K electron wave functions by the variational principle. Proc. Phys. Soc. **76** (1960), 641–649.

Author's summary: "The variational principle is applied to obtain the Dirac wave functions of the K electrons of heavy atoms, using the method of variable parameters. In formulation, three parameters are used. The Coulomb and spin-spin interactions of the two K electrons are taken into consideration as well as the effect of the rest of the atom. A numerical example for mercury, $Z=80$, is given. Its K absorption edge is calculated and relativistic wave functions obtained, using one variable parameter. The results justify the use of screened hydrogenic wave functions although the screening constant (~ 0.5) is somewhat higher than that given by Slater's rules. Calculations using two variable parameters are also given and the total energy is found to have a saddle point and not a true minimum. This is briefly discussed in the light of the hydrogen atom."

5407:

Perkins, D. H. Observations on cosmic ray 'jet' interactions in nuclear emulsions. Progress in elementary particle and cosmic ray physics, Vol. 5, pp. 257–363. North-Holland, Amsterdam; Interscience, New York; 1960.

The author provides a review of experimental data on high-energy nuclear interactions in emulsions specifically related to the so-called "jets". These are formed through the collision of ultra high energy cosmic rays at altitudes well in excess of 40,000 ft. Details of the method of analysis of jets are given as well as a comparison between the experimental results and current theories on the subject.

H. Messel (Sydney)

5408:

Minami, Shigeo. *s*-wave pion-nucleon interaction. Progr. Theoret. Phys. **23** (1960), 887–895.

Author's summary: "Pion-nucleon scattering in the limit of low energy is investigated in order to see some characteristic property of pion-nucleon interaction in the nucleon core. It is pointed out that the value of coupling constant in nucleon core ought to be reduced in appearance to $f=(\mu/2M)g$ in spite of the fact that its value in the neighborhood of pion cloud is g. Moreover, some attempt to eliminate the divergence included in the dispersion relation is made on the basis of the above result, and the experimental results for s-wave phase shifts can be explained satisfactorily."

5409:

Moszkowski, S. A.; Scott, B. L. Nuclear forces and the properties of nuclear matter. Ann. Physics **11** (1960), 65–115.

The authors state that "of all the calculations which have heretofore been made only that of Brueckner and

Gammel seems to be free of gross approximations. Their calculation is, however, quite complicated". Quoting from another place in the paper: "The purpose of this paper is to try to understand these interesting results in a fairly simple way." The work of Gomes, Walecka, and Weisskopf is in the same category. One interesting result of the present paper is that the work of Gomes, Walecka, and Weisskopf is in error.

Continuing the last quote above: "The essential thing is that the nuclear forces are basically attractive at moderately low energies. Although they apparently turn strongly repulsive at short distances, the attraction at larger distances more than compensates this repulsion. In other words, the repulsion effectively cancels a part of the attraction. Now we cannot treat the entire interaction by perturbation theory. However, suppose we throw away the repulsion and that part of the attraction which is cancelled." The "cancellation-distance" is found from ordinary nucleon nucleon scattering, the idea being that the contribution to the Brueckner t-matrices coming from the potential inside the cancellation distance is not much affected by the presence of the other particles.

The numerical results compare well with the results of Brueckner and Gammel where comparison is possible. The authors neglect the tensor and spin orbit terms in the potential; Brueckner and Gammel did not; the authors state that the inclusion of these terms contributed to the complexity of the Brueckner-Gammel calculations.

J. L. Gammel (Los Alamos, N.M.)

5410a:

Kretzschmar, Martin. Gruppentheoretische Untersuchungen zum Schalenmodell. I. Die Mathematische Theorie des Hamilton-Operators. Z. Physik **157** (1960), 433–456. (English summary)

5410b:

Kretzschmar, Martin. Gruppentheoretische Untersuchungen zum Schalenmodell. II. Zum Problem der Translationsinvarianz. Z. Physik **158** (1960), 284–303. (English summary)

The theory of nuclear structure, considered as a non-relativistic many-body problem, is complicated by the fact that a nucleus has no natural fixed point, or center of symmetry. It is difficult for analytical reasons to introduce the center of mass as such a fixed point, and so to separate the internal motions of the nucleons from the external motion of the nucleus as a whole. When the Schrödinger equation is solved by referring it to an arbitrary fixed reference system it is difficult to eliminate unwanted solutions which involve oscillatory motions of the center of mass ("spurious states"). This problem has been studied earlier by direct methods but a general theory has been lacking. The author develops such a theory for the shell model with harmonic oscillator forces and LS-coupling, making maximal use of group-theoretical symmetry considerations.

I. In the first paper spin and isospin variables, and the Pauli principle, are omitted for simplicity. The usual hamiltonian function for a system of oscillators bound to a (fictitious) center is split into the sum of a member representing the internal motions together with a remainder. The symmetry groups of the internal and total hamiltonian operators are introduced as the complex unitary groups of $3(A-1)$ and $3A$ dimensions, respectively, where A is the number of nucleons. The analysis depends on the consideration of the subgroups of these groups and their factorization into direct products of rotation, permutation, and unitary groups. The work is rather complicated, but detailed references to the literature are given. It leads at one step to an interpretation of the distinction between "physical" and "spurious" states in terms of the mathematical concept of an inner plethysm.

II. The second paper is an extension of the first, to include consideration of the spin and isospin variables, the Pauli principle, and the translation invariance of the hamiltonian operator. The connections with Wigner's theory of supermultiplets and with the shell model are examined. Explicit considerations for the ground states of nuclei with $A \leq 18$ are given, as well as for some excited states. The nature of the "spurious states" is examined and it is shown how some of their properties can be determined. The relation between this theory and that developed by J. P. Elliott [Proc. Roy. Soc. London. Ser. A **245** (1958), 562–581; MR **20** #697] on the connection between the nuclear shell and collective models is considered, with the conclusion that the latter work can be incorporated almost directly into the present theory.

E. L. Hill (Minneapolis, Minn.)

5411:

March, N. H.; Murray, A. M. Electronic wave functions round a vacancy in a metal. Proc. Roy. Soc. London. Ser. A **256** (1960), 400–415.

A calculation of the wave functions round a vacancy in a metal has been carried out for the simple model of a negative charge embedded in a free-electron gas. Comparison is made between these results for a finite metal, existing Thomas-Fermi calculations and a self-consistent von Weizsacker calculation, also reported here.

D. F. Mayers (Oxford)

5412:

Dyhne, A. M. Quantum transitions in the adiabatic approximation. Ž. Èksper. Teoret. Fiz. **38** (1960), 570–578 (Russian. English summary); translated as Soviet Physics. JETP **11**, 411–415.

Author's summary: "The probabilities of quantum transitions in the discrete spectrum have been found in the adiabatic approximation assuming a very simple time dependence of the Hamiltonian. The time behavior of the adiabatic invariants has been examined on the example of the classical linear oscillator."

5413:

Sakurai, J. J. Theory of strong interactions. Ann. Physics **11** (1960), 1–48.

In this paper the author presents, in qualitative form, a theory of strong interactions which is based on a fundamental principle. The principle is the invariance of physical laws under "local" gauge transformations which go with the three conservation laws of baryon number, hypercharge, and isotopic spin. Following C. N. Yang and R.L. Mills [Phys. Rev. (2) **96** (1954), 191–195; MR **16**, 432], it is seen that such invariance is possible only if there exist 3 new fields which are vectors in Minkowski space, and are respectively a vector and 2 scalars in isospin space. The form of the interaction of such fields with

existing ones is completely prescribed up to a single constant for each new field. It is then assumed that all strong interactions are induced by the interactions of the gauge fields with known fields.

The remainder of the paper is concerned with analyzing many of the known properties of strong interactions in a qualitative way on the basis of the above assumption. It is shown in a fairly convincing way that many features of strong interactions are indeed understandable if these fields exist, and are rather massive (mass$\gtrsim$several pion masses). {In the opinion of the reviewer, this part of the paper is valuable as a guide to further analyses of strong interactions whether or not its theoretical justification can be maintained.}

A theory of this type necessarily must face several problems. One of these is the necessity of treating some known particles, like pions, as bound states, without having any methods for calculating such things systematically. Another is the ambiguity as to which conservation laws are to be treated as fundamental insofar as applying the local gauge invariance is concerned. A third is the problem of the mass of the new fields introduced, since it is sometimes claimed (without proof, to the reviewer's knowledge), that the gauge invariance used to introduce these fields necessarily restricts them to be massless, which would destroy the whole possibility of the author's theory. These problems are known to the author, who is apparently of the opinion that they can be resolved. Several proposals for resolving some of them are made within the paper, but the author would probably agree that the results are inconclusive. In view of the importance of the problems involved, detailed study of them is surely warranted.
G. Feinberg (New York)

5414:

Barbašov, B. M.; Efimov, G. V. Green's function in the fixed-source model of charged scalar mesons. Ž. Èksper. Teoret. Fiz. **38** (1960), 198–200 (Russian. English summary); translated as Soviet Physics. JETP **11**, 145–146.

Authors' summary: "The calculation of the Green's function for a static nucleon interacting with charged scalar mesons is given as an example of a new method of solution which is different from the perturbation method."

5415:

Mahanthappa, K. T.; Mathews, P. M.; Rau, Jayaseetha. Dynamics of a system of spin 1 particles. Phys. Rev. (2) **115** (1959), 478–481.

The equations of motion for the density matrix for a system with three states ("spin particles") are analyzed in terms of the multiple parametrization; a geometric interpretation of the motion is also given.
E. C. G. Sudarshan (Rochester, N.Y.)

5416:

Leal Ferreira, Jorge; Katayama, Yasuhisa. On a non-local electromagnetic model for electron and muon masses. Progr. Theoret. Phys. **23** (1960), 776–786.

Authors' summary: "In a phenomenological way, a non-local electromagnetic interaction with a Pauli term is assumed in order to explain the whole masses of electron and muon. Qualitative discussions are devoted to the properties of form factors on the assumption of similar internal structures for both particles."

5417:

Ikeda, Mineo; Ogawa, Shuzo; Ohnuki, Yoshio. A possible symmetry in Sakata's model for bosons-baryons system. I. Progr. Theoret. Phys. **22** (1959), 715–724.

Authors' summary: "In this paper we study a possible symmetry in Sakata's model for the strongly interacting particles. In the limiting case in which the basic particles, proton p, neutron n and Λ-particle Λ, have an equal mass, our theory holds the invariance under the exchange of p and Λ or n and Λ in addition to the usual charge independence and the conservation of electrical and hypersonic charge.

"From our theory the following are obtained: (a) iso-singlet π_0'-meson state, which is a pseudo-scalar, exists, (b) the spin of Ξ-particle may be $(3/2)^+$ and (c) several resonating states in K- and π-nucleon scattering are anticipated to exist."

5418:

Ikeda, Mineo; Ogawa, Shuzo; Ohnuki, Yoshio. A possible symmetry in Sakata's model for bosons-baryons system. II. Progr. Theoret. Phys. **23** (1960), 1073–1099.

Authors' summary: "In the previous paper we have discussed a possible symmetry among the proton, neutron and Λ-particle in Sakata's model and obtained some physically interesting results in bosons-baryons system. This symmetry is equivalent to the invariance of the theory under transformations of the unitary group $U(3)$ of degree three. We shall study a mathematical structure of our work in more detail."

5419:

Yamaguchi, Yoshio. Classification of composite bosons in the Sakata model. Progr. Theoret. Phys. **23** (1960), 882–886.

Author's summary: "Assuming the Sakata model (p, n, Λ are basic, all other strongly interacting particles are composite particles) and neglecting moderately strong interactions which contribute to N-Λ mass splitting, we find the complete symmetry between three fundamental fields (referred to as global symmetry). Under this global approximation, classification of two baryon pair states—which are supposed to represent physical mesons—is described."

5420:

Ihara, Chiaki. Pion-nucleon interaction, anomalous magnetic moment of nucleon and composite model for pion. Progr. Theoret. Phys. **23** (1960), 1035–1054.

Author's summary: "The effective pion-nucleon interaction and the anomalous magnetic moment of the nucleon are calculated on the basis of the composite model for the pion in which the fundamental interaction is assumed to be an adequate linear combination of scalar, tensor and pseudoscalar, or vector and pseudovector Fermi type couplings. The results are qualitatively in agreement with experiment."

5421:

Yamamoto, Hiroshi. Bound states in four-nucleon coupling. Progr. Theoret. Phys. **23** (1960), 1100–1116.

Author's summary: "An interaction Lagrangian of general four-nucleon coupling is assumed and equations for nucleon-antinucleon and two-nucleon systems in three types of chain approximation are derived from the interaction and solved exactly. Then the mathematical structure of the solutions is studied in various cases, and the regions where the value of the coupling constant must lie in order to give the bound states are obtained in the case of pseudoscalar coupling. Some interesting features are pointed out."

5422:

Blohincev, D. I.; Barašenkov, V. S.; Barbašov, B. M. The structure of nucleons. Uspehi Fiz. Nauk **68** (1959), 417–447 (Russian); translated as Soviet Physics. Uspekhi **2**, 505–525.

This review article summarizes and gives a valuable critical analysis of recent experimental and theoretical work aimed at the clarification of the "structure" of nucleons, which are considered to consist of a central core (containing hyperons, nucleon-antinucleon pairs, K-mesons) and an outer pion-cloud. The discussion centers about the following topics: Methods for studying the structure; Electromagnetic structure of the nucleons and effects connected with this structure; Theoretical attempts at an interpretation of the electromagnetic structure of the central region; The core of the nucleon; The optical model of the nucleon. *P. Roman* (Boston, Mass.)

5423:

Yakovlev, L. G. Calculation of phase integrals in the covariant formulation of the theory of multiple production of particles. Ž. Èksper. Teoret. Fiz. **37** (1959), 1041–1045 (Russian); translated as Soviet Physics. JETP **10** (1960), 741–743.

"A method is proposed for the exact calculation of integrals over momentum space in the covariant statistical theory of the multiple production of particles" but due to an error in the treatment of the limits in multiple integrals (which leads to a closed form!) the final result for the general case is incorrect.

E. C. G. Sudarshan (Rochester, N.Y.)

5424:

Nikišov, A. I. Statistical theory of multiple particle production. Ž. Èksper. Teoret. Fiz. **38** (1960), 509–512 (Russian. English summary); translated as Soviet Physics. JETP **11**, 369–371.

Author's summary: "If equilibrium does not set in during a collision, one can employ the statistical theory for estimating the mean values of the squares of the matrix elements. In this case, one can hope to obtain satisfactory agreement with experiments for the multiplicity, charge state, and momentum distributions irrespective of the charge of the particle."

5425:

Rus'kin, V. I.; Usik, P. A. The significance of strange particles in Fermi's statistical theory. Ž. Èksper. Teoret. Fiz. **38** (1960), 929–933 (Russian. English summary); translated as Soviet Physics. JETP **11**, 669–672.

5426:

Kanazawa, Akira. Foldy transformation in the pion-hyperon system. Phys. Rev. (2) **118** (1960), 1664–1666.

Author's summary: "A unitary transformation, which plays the same role as the Foldy transformation in the pion-nucleon system, is constructed for the case where the pion interacts with both Σ and Λ hyperons through γ^5 couplings. The transformation function and the transformed Hamiltonian are very similar to those of the Foldy transformation, in spite of the complexity of our system in isotopic spin space. The application to practical problems is not considered in this paper."

S. A. Wouthuysen (Amsterdam)

5427:

Treiman, S. B. Weak global symmetry. Nuovo Cimento (10) **15** (1960), 916–924. (Italian summary)

This paper has as one of its aims the incorporation of the $|\Delta T| = \frac{1}{2}$ rule into the $V-A$ theory of universal four-fermion interaction (which the author calls "the Feynman-Gell Mann theory") and involves the introduction of neutral currents along with the usual charged currents. The arbitrariness in the choice of these currents, which are taken to obey the $\Delta S/\Delta Q = +1$ rule, is "removed by a definite choice, patterned after . . . Gell-Mann's model of global symmetry for strong baryon-pion interactions". While both global symmetry and the rationale of "removing" the arbitrariness in this fashion are subject to question this scheme provides a convenient choice of the undetermined weak interaction Hamiltonian; the model is consistent with the "presently known properties of Σ, Λ and Ξ decays". *E. C. G. Sudarshan* (Rochester, N.Y.)

5428:

Hillion, Pierre; Vigier, Jean-Pierre. Application des groupes d'invariance relativistes aux modèles de particules étendues en Relativité restreinte. C. R. Acad. Sci. Paris **250** (1960), 4117–4119.

The group SL_4 (special Lorentz transformations subjected to the condition that all the velocities have the same direction) can be interpreted either as a translation of axes (gauge transformation), or as a translation of the physical system (kinetic transformation). On the other hand, because of the Thomas precession, the group GL_4 (general Lorentz transformations) allows both interpretations if and only if the properties of the physical system under consideration are not affected by an acceleration. In particular, both interpretations cannot be equivalent for extended particles, since accelerations may modify their internal stresses.

The authors thereby infer that their theory of extended elementary particles (references to earlier work are supplied at the end of the paper) should be invariant under the following groups: SL_4 (kinetic transformations, parallel to the four-velocity of the system), GL_4 (gauge transformations) and an internal group, related to the structure of the particle. This last group is isomorphic to the group $R_3{}^*$ of complex orthogonal transformations.

A. Peres (Haifa)

5429:

Hillion, Pierre; Vigier, Jean-Pierre. Forme possible des fonctions d'ondes relativistes associées au mouvement et à

la structure des particules élémentaires, au niveau nucléaire. C. R. Acad. Sci. Paris **250** (1960), 4295–4297.

The results of the previous paper [see preceding review] are applied to the derivation of relativistic wave functions for elementary particles. These wave functions belong to the irreducible representations of the direct product of SL_4 and R_3^*.

A. Peres (Haifa)

5430:

Grin′, Yu. T.; Drozdov, S. I.; Zareckiĭ, D. F. Green's function for odd nuclei. Ž. Èksper. Teoret. Fiz. **38** (1960), 222–228 (Russian. English summary); translated as Soviet Physics. JETP **11**, 162–166.

Authors' summary: "The techniques of many-body theory are applied to a study of pair correlations in finite systems with an odd number of particles. The Green's function is found, and perturbation theory developed."

5431:

Watson, Kenneth M. Quantum mechanical transport theory. I. Incoherent processes. Phys. Rev. (2) **118** (1960), 886–898.

This paper is concerned with developing a general formalism which is adequate for the description of the transport of particles through a medium. The way in which inelastic scatterings may be described in terms of an essentially classical transport equation is developed, as is also the introduction of an index of refraction for the case of elastic scattering. The importance of interferences of waves from nearby scatterers in the case of small excitation of the medium is emphasized. The mathematical techniques used are adaptations of multiple scattering methods developed by the author in many other papers, but the present paper is reasonably self contained.

M. L. Goldberger (Princeton, N.J.)

5432:

Gross, Eugene P. Quantum theory of interacting bosons. Ann. Physics **9** (1960), 292–324.

Author's summary: "Some qualitative features of the ground state of a system of interacting bosons are discussed using wave functions suggested by the semiclassical theory of boson wave fields. For the case where one deals with weak repulsions, one is lead to a variational extension of Bogolyubov's work. A finite fraction of the N particles occupies the zero momentum single particle state, and the dynamic correlations are described by pair excitations. When attractive forces play a decisive role, two cases are found. In one case a finite fraction of the particles occupies a single particle state, which is now periodic in space. The dynamic correlations are described as a generalization of pair excitations which is different in character for excitation moments of the order of the inverse of the range of the attractive forces. The single particle state and dynamic correlations are codetermined in a systematic way. The approximate ground state shows long range order which is destroyed at finite temperatures. A second case where attractions are important is the solid state of the boson system. The ground state has the property that of the order of N orthogonal single particle states are occupied, each with an average of approximately one particle."

R. Arnowitt (Syracuse, N.Y.)

5433:

Fujita, J. Pseudo-cluster expansion. Nuclear Phys. **14** (1959/60), 648–660.

A strongly-interacting, many-body system has its wave function Ψ for the ground state related to an asymptotic Φ by the model operator M by $\Psi = M\Phi$, a velocity-dependent short-range correlation being thus isolated. M is defined by properties related to the hard-sphere separation of any pair of particles. A pseudo-cluster expansion is given analogous to the Ursell-Mayer cluster-expansion method. Possible applications are discussed.

C. Strachan (Aberdeen)

5434:

Gor′kov, L. P. Theory of superconducting alloys in a strong magnetic field near the critical temperature. Ž. Èksper. Teoret. Fiz. **37** (1959), 1407–1416 (Russian. English summary); translated as Soviet Physics. JETP **10** (1960), 998–1004.

Green's function methods are used to obtain the Landau-Ginzberg phenomenological equations for superconducting alloys in the "London" region (close to the critical temperature), starting from the B.C.S. theory, modified by impurity scattering of the electrons. This reviewer does not claim to understand the derivation.

J. M. Blatt (Murray Hill, N.J.)

5435:

Thouless, David J. Perturbation theory in statistical mechanics and the theory of superconductivity. Ann. Physics **10** (1960), 553–588.

Summation of the ladder diagrams of the perturbation theory for the grand canonical partition function shows that this series fails to converge below a critical temperature T_c, equal to that obtained in the Bogoliubov theory of superconductivity. A "separable" interaction allows a more detailed evaluation of the theory in the neighbourhood of T_c. This yields a weak $(T-T_c)^{-1/2}$ singularity in the specific heat. The reviewer doubts whether this result applies to metals, since separability of the interaction is an extremely restrictive assumption.

J. M. Blatt (Murray Hill, N.J.)

5436:

Cohen, Michael. Relation between inelastic neutron scattering and thermodynamic functions of liquid helium. Phys. Rev. (2) **118** (1960), 27–41.

The effects of the temperature dependence of the excitation spectrum of liquid helium are studied. It is assumed that there is a two-body interaction between the rotons that gives rise to a shift in their energies and makes their lifetimes finite. The expansion of the free energy in terms of linked graphs is used, and only those terms which are most important at very low temperatures are retained. It is found that the thermodynamic functions are simply related to the excitation spectrum obtained by neutron scattering only in the case of weak interactions. Since the line-widths are found experimentally to be comparable with the line-shifts, the interaction cannot in fact be regarded as weak. The normal fluid density is also calculated, and corrections to the simple theory are found that do not vanish even for weak interactions.

D. J. Thouless (Birmingham)

5437:

Gor'kov, L. P. The critical supercooling field in superconductivity theory. Ž. Èksper. Teoret. Fiz. **37** (1959), 833–842 (Russian); translated as Soviet Physics. JETP **10** (1960), 593–599.

The critical supercooling field is calculated by studying the Green's functions for a normal metal in the presence of a uniform magnetic field. When the field is less than the critical supercooling field, the normal state is unstable for infinitesimal departures from normality. The calculated critical supercooling field is compared with the thermodynamic critical field, for which the normal state and the uniform superconducting state have the same free energy. The critical supercooling field is less than the thermodynamic critical field for a Pippard metal, whose coherence length is greater than the penetration depth, and so there is a region of metastability where the normal metal can exist although it is not thermodynamically stable. For London metals the critical supercooling field is greater, since the surface energy between the normal and superconducting phases is negative. The calculations are carried out at zero temperature, but the Ginzburg-Landau theory is used to show that the ratio between the two critical fields is only slightly different near the critical temperature. The results are compared with experiment, and the agreement is moderately satisfactory.

D. J. Thouless (Birmingham)

5438:

Èliašberg, G. M. Interactions between electrons and lattice vibrations in a superconductor. Ž. Èksper. Teoret. Fiz. **38** (1960), 966–976 (Russian. English summary); translated as Soviet Physics. JETP **11**, 696–702.

Author's summary: "A perturbation theory is developed for the Green's function in which the Green's function calculated for the superconducting ground state is used as the zero approximation. Dyson equations are written down from which the electron Green's function can be determined. Interaction between electrons and phonons is not assumed to be small. The spectrum and the damping of the excitations are calculated."

RELATIVITY

See also 5428, 5429.

5439:

v. Krzywoblocki, M. Z. On the general form of the special theory of relativity. Acta Phys. Austriaca **13** (1960), 387–394.

The author investigates the transformation equations that will leave a four-dimensional line element invariant that looks like the Minkowski line element, except that c is to be a function of the coordinates. To the reviewer it appears that the investigation is neither physically motivated nor mathematically unambiguous: Though the transformation equations appear linear, actually the coefficients themselves are assumed to be functions of the coordinates; the transformed function c' is not assumed to be identical with c. *P. G. Bergmann* (Syracuse, N.Y.)

5440:

Bertotti, B.; Plebanski, J. Theory of gravitational perturbations in the fast motion approximation. Ann. Physics **11** (1960), 169–200.

The purpose of the paper is to find equations of motion using k=grav. constant as a parameter in the expansion of the gravitational equations. The method is a generalization of the EIH method which uses only c^{-1} as the parameter of expansion. Although many papers have been written on this subject this is the first which advances the calculations to second order and gives prescriptions for the use of the approximation procedure generally. This procedure can be applied to any non-linear theory as well. However, as the authors remark: "Metrical equations we get are ... 'paper and pencil stuff'!"

L. Infeld (Warsaw)

5441:

Plebanski, Jerzy. Electromagnetic waves in gravitational fields. Phys. Rev. (2) **118** (1960), 1396–1408.

The author assumes that at a time $X_0=-\infty$ a plane wave with given wavelength direction, and polarisation is propagated towards a given gravitational field (not influenced by the incoming wave). At $X_0=+\infty$ the wave arrives at "the other side" of infinity. The question which the author answers is how will the numbers characterizing the incoming wave change under the influence of the gravitational field. *L. Infeld* (Warsaw)

5442:

Klein, O. On the treatment of the gravitational field in connection with the generally relativistic Dirac equation. Ark. Fys. **17**, 517–520 (1960).

The independent variation of the metric and affinity in the Lagrangian of general relativity is of use is putting the gravitational field in canonical form. For the case in which a spinor field is coupled to gravitation, it is here conjectured that it may, correspondingly, be simpler to consider the spinor field γ^μ and "spinor affinity" Γ_μ as independent. In this paper, a formulation of the gravitational field itself is therefore given in which the γ^μ and Γ_μ are to be the basic independent variables, rather than the metric, so that the curvature scalar is to be expressed in terms of these quantities. The resulting field equations and their relation to the usual formulation are discussed. [Further work on these lines may be found in B. E. Laurent, Ark. Fys. **16** (1960), 263–278; MR **22** #2424.]

S. Deser (Waltham, Mass.)

5443:

Černov, Yu. P. Stationary rotation of cosmic gaseous masses in general theory of relativity. Dokl. Akad. Nauk SSSR **129** (1959), 762–765 (Russian); translated as Soviet Physics. Dokl. **4** (1960), 1230–1234.

The relativistic equations are formulated for the steady state rotation of a gaseous mass, filling the entire space, about an axis of symmetry. The Einstein-Infeld-Hoffmann method of successive approximations [Einstein, Infeld and Hoffmann, Ann. of Math. (2) **39** (1938), 65–100] is used to find the solution for a prescribed mass-density. *C. Gilbert* (Newcastle-upon-Tyne)

5444:

Peres, Asher; Rosen, Nathan. Gravitational radiation damping of nongravitational motion. Ann. Physics **10** (1960), 94–99.

The rate of working of non-gravitational forces in a material system is defined. It is shown that in linearized gravitation theory this rate is equal, under conditions which are stated, to the rates, measured by the energy pseudo-tensor, at which gravitational energy is radiated by the system. *F. A. E. Pirani* (London)

5445:

Mickevič, N. V. On consequences of the condition of invariance of the Lagrangians in the general covariant field theories. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 3, 63–70. (Russian)

The author discusses the consequences of the condition of the invariance of the Lagrangians in the general covariant field theories. Starting from the condition $\delta L=0$, L=Lagrangian, the author introduces a transformation of coordinates $x'^{\alpha}=x^{\alpha}+\delta x^{\alpha}$, which results in the variations of the applied potential fields, A_B, of the form: $\delta A_B=a_{B,\alpha}{}^{\beta}\,\partial\delta x^{\alpha}/\partial x^{\beta}$; this is followed by a discussion of the properties of the resulting formulas. Next item discussed is the tensors of the gravitational fields, satisfying the usual condition of $T_t{}^{\mu\nu}=0$. The author formulates a "principle of simplicity" which states that: (1) L should be a function of the least number of variables, and (2) L should be a function of the lower order of these variables. An application of this principle to a gravitational field and a derivation of some conclusions from this procedure closes the paper.

M. Z. v. Krzywoblocki (E. Lansing, Mich.)

5446:

Surdin, Maurice. Une expérience destinée à vérifier la théorie de la relativité généralisée. C. R. Acad. Sci. Paris **250** (1960), 299–301.

This note suggests the arrangement of an experiment designed to test the validity of the general theory of relativity. It is to depend on the effect of gravitation on two clocks situated at different altitudes, these clocks to be of the Maser type by means of which very weak changes in frequency may be detected. [See also R. S. Badessa, R. L. Kent and J. Nowell, Phys. Rev. Lett. **3** (1959), 79–80; and H. Yilmaz, ibid., 320–321.]

H. Rund (Durban)

5447:

Knapecz, Géza. Über die Erhaltungssätze des allgemein-relativistischen Lagrange-Formalismus. Ann. Physik (7) **6** (1960), 44–54.

The author classifies all conservation theorems, and the quantities that are conserved, according to the following properties: (a) strong conservation laws (identities) vs. weak conservation laws (which hold only if the field equations are satisfied); (b) whether the conservation law is covariant or not; and (c) whether it exists by virtue of the invariant character of the action principle alone, or whether the law exists only if the Lagrangian contains a term (or terms) independent of all field variables save the metric tensor and its derivatives. Additionally, a distinction is made, in the case of covariant conservation laws, as to whether the covariant differential operator is a pure derivative ("true" conservation law) or whether it contains terms involving Christoffel symbols (pseudo-conservation). With the help of this extensive classification scheme, the author derives all conservation laws and compares them with special forms found in the literature, particularly those of Goldberg [Phys. Rev. (2) **89** (1953), 263–272; MR **14**, 805], Bergman [ibid. **112** (1958), 287–289; MR **20** #5677] and Mizkjewitsch [same Ann. (7) **1** (1958), 319–333; MR **20** #722].

P. G. Bergman (Syracuse, N.Y.)

5448:

Roche, Claude. Sur la quantification du champ à l'approximation linéaire en théorie de Jordan-Thiry. C. R. Acad. Sci. Paris **250** (1960), 3128–3130.

The author considers a linearized form of the Jordan-Thiry unified field theory [A. Lichnerowicz, *Théories relativistes de la gravitation et de l'électromagnétisme*, Masson, Paris, 1955; MR **17**, 199; pp. 180–214] and obtains, by standard methods, commutation relations for the field variables. These commutation relations are compatible with the gauge conditions imposed on the field (harmonic coordinates). As pointed out by the author, the choice of the commutation relations is partly arbitrary: compare with A. Capella, same C. R. **250** (1960), 2140–2142 [MR **22** #3555].

A. Peres (Haifa)

5449:

Tonnelat, Marie-Antoinette; Bouche, Liane. Quelques remarques sur le schéma matière pure dans une théorie asymétrique du champ de gravitation pure [d'après D. W. Sciama et O. Costa de Beauregard]. C. R. Acad. Sci. Paris **250** (1960), 4289–4291.

It is shown that the reviewer's non-symmetric theory of the pure gravitational field [Proc. Cambridge Philos. Soc. **54** (1958), 72–80; MR **20** #727] implies that the streamlines of a perfect fluid ($T^{\mu\nu}=\rho v^{\mu}v^{\nu}$) are geodesics of the Riemannian space with metric $g^{\underline{ij}}$. These streamlines are not geodesics of the metric $g_{\underline{ij}}$, but deviate from them by an amount which, in first approximation, agrees with a heuristic formula occurring in Costa de Beauregard's theory [same C. R. **250** (1960), 984–986; MR **22** #2433] of the gravitational effects of spin.

D. W. Sciama (Ithaca, N.Y.)

5450:

Rumer, Yu. B. Action as a space coordinate. X. Ž. Èksper. Teoret. Fiz. **36** (1959), 1894–1902 (Russian); translated as Soviet Physics. JETP **9**, 1348–1353.

The five-dimensional field theory treated in this paper employs a symmetric fundamental tensor $G_{\mu\nu}$, where $G_{ik}=g_{ik}+(1+\chi)g_ig_k$, $G_{i5}=(1+\chi)g_i$, $G_{55}=1+\chi$. g_{ik} is the gravitational potential, g_i is proportional to the electromagnetic potential, and χ is a scalar field. The five-dimensional space is topologically closed in the fifth coordinate ($x^5=$) S, the action; so that all the $G_{\mu\nu}$ are periodic functions of S, the period being Planck's constant h. In his previous work *Issledovaniya po 5-optike* [Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1956; MR **20** #2975], the author meant general covariance to be covariance under the general transformations $x^{\mu\prime}=x^{\mu}+f^{\mu}(x^{\nu})$, limited by the condition that all f^{μ} must be periodic functions of x^5 of period h. The difficulties encountered in formulating correct equations for free electrons, or electrons in an external electromagnetic field, are here overcome by imposing a considerable restriction on the transformations to be allowed in the theory: the f^{μ} above are now permitted to depend only on the x^k, but not on S.

H. A. Buchdahl (Hobart)

ASTRONOMY

See also 5169, 5333.

5451:

Vening Meinesz, F. A. The outside gravity field up to great distance from the earth. Nederl. Akad. Wetensch. Proc. Ser. B **62** (1959), 109–114.

The usual expressions for the second and fourth harmonics of the earth's gravitational potential are derived as a function of the oblateness of the earth, under the assumption that it is an exact ellipsoid of revolution. From these are derived expressions for the gravity vector at any point in space.

The effects of disturbing masses within the earth on the potential at any point in space are found, using Stokes' method. The variation in the intensity of gravity at a point whose radius is $1/u$ is found by averaging the free-air anomalies of gravity over the surface of the earth, using a weighting function

$$G_\rho = 2u^{5/2}\frac{d}{du}\left(u^{1/2}\sum_2^\infty P_n u^n\right),$$

where the P_n are surface zonal harmonics expanded around the axis through the point. [See W. A. Heiskanen and F. A. Vening Meinesz, *The earth and its gravity field*, McGraw-Hill, New York, 1958; pp. 63, 64.]

J. A. O'Keefe (Chevy Chase, Md.)

5452:

Brouwer, Dirk. Comments on general theories of planetary orbits. Proc. Sympos. Appl. Math., Vol. 9, pp. 152–166. American Mathematical Society, Providence, R.I., 1959.

An elementary discussion of the problem of calculating perturbations by the method of variation of the elements stresses the D'Alembert characteristic, namely, that the Fourier series in $\sin k\phi$ for several important angular variables have factors of at least the kth power of the orbital eccentricity in the coefficients. If the eccentricity is small, this characteristic serves an important purpose in limiting the danger from resonances between high harmonics of the principal frequencies.

Other methods of dealing with the resonances are the following. (a) Inclusion of power series in the time (secular perturbations) along with the usual periodic terms. These compel the redevelopment of the series after several centuries. (b) Numerical integration of the elements.

The variation of the elements may be abandoned in principle, and replaced by perturbations in position superposed, after the manner of Hansen, on a steadily moving ellipse of fixed shape and size.

It is suggested that the problem of finding planetary theories valid for an indefinite length of time may be solved by the method of variation of the elements, using a sequence of canonical transformations after the manner of Delaunay, to eliminate the periodic terms. The Gauss equations for the time derivatives of the elements are to be used with a perturbation function expressed in polar coordinates.

J. A. O'Keefe (Chevy Chase, Md.)

5453:

Littlewood, J. E. The Lagrange configuration in celestial mechanics. Proc. London Math. Soc. (3) **9** (1959), 525–543; addendum, **10** (1960), 640.

In this paper the author extends a discussion of the motion of a massless particle in the vicinity of either one of the two Lagrangian equidistant equilibrium points of the restricted three-body problem presented in a previous paper [same Proc. (3) **9** (1959), 343–372; MR **21** #7789].

As the equilibrium point is stable when the mass-ratio of two finite masses satisfies a certain inequality, the nearby motion is of oscillatory character. If the initial deviation of the particle from the equilibrium point is very small, the motion may be discussed by linear theory with omission of the terms of second and higher orders. The solution is then a superposition of two modes of simple oscillation with mean motions λ and μ, say, λ, μ being functions of the mass-ratio of two finite masses. Now, if the equations of motion are discussed in their full forms, the solution may be expressed by doubly periodic Fourier series of the form

$$\sum_{j_1,j_2} C_{j_1,j_2} {}^{\sin}_{\cos}[(j_1\Lambda+j_2\mathrm{M})t+\text{constants}],$$

$\Lambda=\lambda+O(\varepsilon)$, $\mathrm{M}=\mu+O(\varepsilon)$, ε being a small number appropriately chosen in relation to the initial deviation. The mean motions Λ and M depend on the amplitudes as well as the mass-ratio, a usual feature of non-linear oscillations. Questions concerning the convergence of these series present an important but difficult problem.

In the present and previous papers the author approaches this problem by discussing "asymptotic" integrals of the motion, that is, approximate integrals which tend to exact ones with $\varepsilon\to 0$. When ε is finite (but small), asymptotic integrals are not exact integrals in two respects: (1) If I is an asymptotic integral, then we may write $I=\text{constant}+E$, where E is an error term which is not constant during the motion; (2) this relation holds for only a limited interval of the time, say $0\leq t<T$. The author establishes explicit expressions of $O(E)$ and T as functions of ε.

The previous paper contains a discussion of two of the integrals; this paper gives all four integrals and the solution itself. The expressions for $O(E)$ and T in the solution are found to be

$$O(E) = O[\exp(-\mathbf{a}_6\varepsilon_1^{-1/2}|\log\varepsilon_1|^{-3/4})],$$
$$T = \mathbf{a}_3[\max(a,b)]^{-1}\exp(\mathbf{a}_4\varepsilon_1^{-1/2}|\log\varepsilon_1|^{-3/4}),$$

where εa, εb are two amplitudes (taken positive) of the linear theory solution, and $\varepsilon_1=\max(\varepsilon a,\varepsilon b)$; $\mathbf{a}_3$, $\mathbf{a}_4$, and $\mathbf{a}_6$ are positive constants depending only on the mass-ratio of two finite masses. Though these bounds do not imply the convergence of the solution, the establishment of time bound and error bound is important.

On page 525, $\mathrm{m}_0(1+\mathrm{m}_0)^2=1/27$ should be read $\mathrm{m}_0(1+\mathrm{m}_0)^{-2}=1/27$; in (1.3), on page 526, $\sum_{m=0}^{n-1}\varepsilon^m\psi_m$ should be read $\sum_{m=1}^{n-1}\varepsilon^m\psi_m$.

Gen-ichiro Hori (New Haven, Conn.)

5454:

Popovič, Božidar. Calculation of the vectorial elements of the orbit of a small planet from three heliocentric positions. Glasnik Mat.-Fiz. Astr. Društvo Mat. Fiz. Hrvatske. Ser. II **14** (1959), 153–158. (Russian. Serbo-Croatian summary)

In this short paper the author presents the method for precise calculation of vectorial elements of the orbit of a small planet from three accurately determined heliocentric positions. This method allows the diminution of

the number of calculations in comparison with known Laplace and Gauss methods. As an illustration of this method, one numerical example, concerning the elements of the small planet "Whittemora" [see G. Stracke, *Bahnbestimmung der Planeten und Kometen*, Berlin, 1929] is discussed. *D. P. Rašković* (Belgrade)

5455:

Contopoulos, George. A third integral of motion in a galaxy. Z. Astrophys. **49** (1960), 273–291.

Author's summary: "A formal third integral of motion in a galaxy is found in the form of a series. We begin with the special case when the potential function is given by:

$$2W = \frac{C^2}{r^2} - P\xi^2 - Qz^2 + 2b\xi z^2,$$

where $\xi = r - r_0$. Then the third integral has the form $\Phi = \Phi_0 + b\Phi_1 + b^2\Phi_2 + \cdots$ where Φ_i are polynomials in ξ, z and $R(=d\xi/dt)$, $Z(=dz/dt)$. Similar results are found in the general case when W is a series in ξ and z. The question of the convergence of Φ remains open.

As an application of the new integral we calculate the boundary of the space filled by an orbit in the ξ, z plane; this space is approximately a trapezium. The results are in very good numerical agreement with the shape of two orbits calculated previously by means of an electronic computer." *M. Janet* (Paris)

5456:

Résibois, P.; Prigogine, I. Analytical invariants in *N*-body systems. Acad. Roy. Belg. Bull. Cl. Sci. (5) **46** (1960), 53–60. (French summary)

For a system subject to the usual periodic boundary conditions it is shown that, contrary to a famous statement of Poincaré's, there exists a class of invariants analytical in the strength of the interaction between the particles in the system. These invariants possess a singular Fourier transform, and the Hamiltonian is the only invariant which is analytical in the coupling constant and which has a regular Fourier transform. It is interesting to note that the systems considered here are not metrically indecomposable in the sense of classical ergodic theory. *D. ter Haar* (Oxford)

5457:

Anselone, Philip M. Convergence of Chandrasekhar's method for the problem of diffuse reflection. Monthly Not. Roy. Astr. Soc. **120** (1960), 498–503.

Let $I(\tau, \mu)$, $J(\tau)$ and $\mathfrak{J}(\tau)$ denote the intensity, average intensity and source functions for radiation in a semi-infinite, plane parallel, isotropically scattering atmosphere, with albedo $\varpi_0 \leq 1$ and the only external source due to an incident parallel beam.

Let $I_m(\tau, \mu)$, $J_m(\tau)$ and $\mathfrak{J}_m(\tau)$ denote the corresponding Chandrasekhar approximation [Chandrasekhar, *Radiative transfer*, Oxford Univ. Press, New York, 1950; MR **13**, 136; pp. 80–87]. It is proved that $I_m \to I$, $J_m \to J$ and $\mathfrak{J}_m \to \mathfrak{J}$ uniformly as $m \to \infty$. Error bounds are obtained in the non-conservative case ($\varpi_0 < 1$). *E. Schatzman* (Paris)

5458:

Fowler, William A.; Hoyle, F. Nuclear cosmochronology. Ann. Physics **10** (1960), 280–302.

It is shown how the observed abundance ratios of different isotopes of uranium and thorium can be used to determine a cosmical time-scale. The significance of this time-scale depends on which are the dominant processes leading to the formation and distribution of the heavy elements. Two models of nucleosynthesis are described. In the first, our galaxy has been autonomous since its formation, that is, it has not been contaminated by intergalactic matter. The time-scale then refers to the age of the galaxy, which is calculated to be $15_{-3}^{+5} \times 10^9$ years. In the second model, the galaxy acquired significant quantities of intergalactic material at various times. This intergalactic material contains heavy elements ejected from other galaxies. The time-scale now refers to the rate of expansion of the universe, giving for Hubble's constant the value $11 \pm 6 \cdot 10^9$ years, in good agreement with observation. *D. W. Sciama* (Ithaca, N.Y.)

GEOPHYSICS

See also 5078, 5218.

5459:

Kamenkovič, V. M. Quasi-stationarity of drift currents in the ocean. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1960**, 74–82. (Russian)

5460:

Pfeffer, Richard L. (Editor). ★Dynamics of climate. The proceedings of a conference on the application of numerical integration techniques to the problem of the general circulation, held October 26–28, 1955. Pergamon Press, New York-Oxford-London-Paris, 1960. xv + 137 pp. (1 plate) $6.50.

This volume gives an excellent picture of the situation, as it was in 1955, as regards the application of numerical methods to the problem of the general circulation of the atmosphere. It is perhaps unfortunate that the volume has been so long in appearing but it is none the less welcome in view of the large amount of time and effort currently being devoted to numerical methods of short range weather prediction.

The first seven papers all deal with numerical methods and related topics such as methods of incorporating non-adiabatic effects in model atmospheres, the design of a numerical experiment to study the response of an idealized atmosphere to different conditions of heating and rotation, and the maintenance of zonal flow and of quasi-stationary perturbations of the zonal flow.

The next section contains several papers on the role of rotation and heating in the mechanism of energy release, a description of laboratory experiments which have a bearing on the circulation problem, and two papers on possible causes of climatic fluctuation.

Two papers on radiation transfer follow—the first of which surveys the problem involved in introducing the effects of long-wave radiation into mathematical models of the general circulation. The volume ends with a summary of the informal discussion which took place at the conference. *M. H. Rogers* (Bristol)

5461:

Phillips, Norman A. Numerical weather prediction. Advances in computers, Vol. 1, pp. 43–90. Academic Press, New York, 1960.

This article gives a comprehensive account of the present day state of numerical weather prediction. After a brief section on the problems concerning initial data in making a numerical forecast, the author discusses the basic differential equations governing the motion and then derives both the hydrostatic and geostrophic forecast systems. A section on computational techniques follows and this includes a careful examination of the computational error in marching problems illustrated by a discussion of the linear advection equation. The final section is devoted to a survey of results of extended series of forecasts and there is also a very full list of references. *M. H. Rogers* (Bristol)

5462:

Smagorinsky, J. On the application of numerical methods to the solution of systems of partial differential equations arising in meteorology. Frontiers of numerical mathematics, pp. 107–125. University of Wisconsin Press, Madison, Wis., 1960.

Without going into any mathematics, the author gives a general survey of the problems encountered in numerical weather prediction. *M. H. Rogers* (Bristol)

5463:

Malkevič, M. S. On the effect of non-orthotropicalness of an underlying surface on scattered light in the atmosphere. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1960**, 440–448. (Russian)

5464:

Friedlander, S. K. Similarity considerations for the particle-size spectrum of a coagulating, sedimenting aerosol. J. Meteorol. 17 (1960), 479–483.

Author's summary: "An explanation is offered for the similarities observed experimentally among size distributions of natural aerosols measured at different places and times. This explanation is based on a similarity transformation of the equation describing the kinetics of a coagulating, settling aerosol. On dimensional grounds, an approximate shape is derived for the upper end of the spectrum which compares well with the available experimental data. An estimate is given for the rate at which matter is transferred from the lower to the upper end of the spectrum."

5465:

Rosenthal, Stanley L. A simplified linear theory of equatorial easterly waves. J. Meteorol. 17 (1960), 484–488.

Author's summary: "If the perturbation of the zonal wind component and the Coriolis term which arises in the zonal equation of motion as a result of vertical motions are neglected, the linearized vorticity equation and the continuity equation (when written in pressure coordinates) become a complete set for the meridional wind and vertical motion perturbations. This set is solved for a class of easterly waves which reach their maximum intensity at the equator and dampen poleward.

"The theoretical streamlines and the theoretical field of divergence both agree quite well with their empirical counterparts. On the other hand, the theoretical isotachs are somewhat distorted and the theoretical phase speed is a bit low."

5466:

Malkevič, M. S. An approximative method for calculation of the horizontal variations in the albedo of an underlying surface in problems on the scattering of light in the atmosphere. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1960**, 288–298. (Russian)

5467:

Bagdoev, A. G. Determination of pressure in the neighborhood of a front. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1960**, 653–657. (Russian)

5468:

Gol'cman, F. M. Frequency theory of interference systems. I, II. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1960**, 7–23, 209–222. (Russian)

5469:

Nemčinov, S. V. On solution of the equation for prognosis of the field of atmospheric pressure. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1959**, 1821–1830. (Russian)

5470:

Čžu, Yun'-Ti. On calculation of the dynamical influence of mountain masses in the non-linear problem of long-term prognosis of meteorological elements. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1959**, 1807–1820. (Russian)

5471:

Bulaševič, Yu. P.; Šulyat'ev, S. A. Optimal conditions for continuous activated coring. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1960**, 253–262. (Russian)

5472:

Duclaux, F. Séismométrie théorique. Mémor. Sci. Phys. **64** (1960), 129 pp.

This book is a laboratory manual on the use, adjustment and checking of electromagnetic seismographs, as well as seismographs with mechanical amplification.

It contains also the elements of general theory of these apparatuses with detailed discussion of Galitzin and Wenner types. Written from the point of view of practical applications and every-day work, this book will be very useful not only for the students of seismography but also for the members of seismographic stations. *E. Kogbetliantz* (New York)

5473:

Gal'perin, E. I.; Frolova, A. V. Azimuthal-phase correlation of elliptically polarized seismic waves. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1960**, 195–208. (Russian)

5474:

Pod"yapol'skiĭ, G. S. An approximate expression for the displacement in the neighborhood of a basic front in the case of a small angle between the ray and the boundary of partition. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1959**, 1761–1773. (Russian)

5475:

Yanovskaya, T. B. Investigation of dispersing surface waves in the neighborhood of a minimum of the group velocity. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1959**, 1753–1760. (Russian)

5476:

Kogan, S. Ya. On determination of the energy of seismic waves of arbitrary shape. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1960**, 644–652. (Russian)

5477:

Strahov, V. N. Integral methods of interpretation of the anomalies ΔZ of constant sign. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1960**, 520–529. (Russian)

OPERATIONS RESEARCH, ECONOMETRICS, GAMES

See also A4647, A4898, 5078, 5116.

5478:

Arrow, Kenneth J.; Karlin, Samuel; Suppes, Patrick (Editors). ★Mathematical methods in the social sciences, 1959. Proceedings of the first Stanford Symposium. Stanford Mathematical Studies in the Social Sciences, IV. Stanford University Press, Stanford, Calif., 1960. viii + 365 pp. $8.50.

The 23 papers are organized into three sections: economics, management science, and psychology. Each paper is receiving an independent review in MR.

5479:

Mori, Kei. Graphical, analog, and digital solution of Dr. Goodwin's nonlinear business cycle model. Proc. Fac. Engrg. Keio Univ. **11** (1958), 102–122.

In the Goodwin business cycle model nonlinearity is due to the assumption of a floor and a ceiling as constraints upon the variation of the national product. The present article gives interesting numerical results generated by this model. *T. Haavelmo* (Oslo)

5480:

Vellando, G. Arnaiz; Lasuen, J. R. Economic significance of the coefficients in input-output-analysis. Estadíst. Española No. 5, 27–33 (1959). (Spanish. English summary)

A discussion of the economic significance of the entries in the Leontief matrix and its inverse. *H. Rubin* (E. Lansing, Mich.)

5481:

Sinden, Frank W. The replacement and expansion of durable equipment. J. Soc. Indust. Appl. Math. **8** (1960), 466–480.

Formulates the problem of determining the optimal times for the replacement of a single facility whose cost function depends on the time of acquisition, the time elapsed, and the time at which it will be replaced. Special cases are explored. The technique is straightforward minimization. *M. J. Beckmann* (Providence, R.I.)

5482:

Rashevsky, N. Further contributions to the mathematical biophysics of automobile driving. Bull. Math. Biophys. **22** (1960), 257–262.

From the author's summary: "The discussions of a previous paper [same Bull. **21** (1959), 299–308; MR **21** #6279] are generalized by considering that the angular direction error made by the driver, as well as the driver's reaction time, are not constant but are randomly distributed." *G. Newell* (Providence, R.I.)

5483:

Greenberg, Harold; Daou, Arthur. The control of traffic flow to increase the flow. Operations Res. **8** (1960), 524–532.

It appears that inside the Holland Tunnel between New York and New Jersey there is a bottleneck where the capacity is somewhat lower than the entrance capacity. The flow of traffic between the entrance and this bottleneck is unstable and it oscillates between an overcapacity flow rate and essentially zero flow. This paper presents some of the experimental data describing this situation and how the instability was eliminated by restricting the maximum rate at which cars were allowed to enter the tunnel. These controls were found to give a net increase in the average flow. No satisfactory theory exists to explain this instability, however. *G. Newell* (Providence, R.I.)

5484:

Chiassino, Giuseppe. Sulla rappresentazione analitica della vita media. Statistica. Bologna **20** (1960), 204–215.

Der Verfasser diskutiert für die Berechnung der mittleren Lebenserwartung $\bar{E}_x$ eines x-Jährigen die Formeln $\bar{E}_x = ks^x g^{c^x}$, $\bar{E}_x = A(B-Cx)^p$, $\bar{E}_x = ks^x + ax + b$. *W. Saxer* (Zürich)

5485:

Barnhart, E. Paul. Continuance functions. Soc. Actuar. Trans. **11** (1959), 649–722 (1960).

The author gives a theoretical basis for the mathematical graduation of continuance data for both elementary and combined benefits. He demonstrates methods of utilizing the theory to derive claim costs for a wide variety of benefits as the evaluation of claim costs for multiple medical benefits. He defines the force of termination by the functions $\pi^{(t)} = a/(a+t)$ or $l/(\lambda - t)$ and calculates the continuance integrals. He discusses various graduation problems and gives many statistics and practical examples for his theory. *W. Saxer* (Zürich)

5486:

Bussi, Carlo. Considerazioni sulle applicazioni del metodo della programmazione lineare. Atti Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat. **94** (1959/60), 202–210.

An elementary account of linear programming methods which stresses economic applications.

G. Tintner (Ames, Iowa)

5487:

Aronszajn, N. Sur un problème de transport. Bull. Sci. Math. (2) **83** (1959), 9–20.

The author presents a method for solving the transportation problem of linear programming. The method is based on four theorems, stated but not proved, that are commonly known to present workers in the field. The paper is historically important, because it was written in 1940 but was not published until recently because of military security restrictions. Maurice Fréchet states in the preface that M. Divisia formulated the problem for attention of Émile Borel's wartime research group, in connection with a problem of railroad operations. The work of Divisia and Aronszajn is apparently prior to that of Hitchcock (1942) but at about the same time as that of Kantorovich.

M. M. Flood (Ann Arbor, Mich.)

5488:

Flood, Merrill M. An alternative proof of a theorem of König as an algorithm for the Hitchcock distribution problem. Proc. Sympos. Appl. Math., Vol. 10, pp. 299–307. American Mathematical Society, Providence, R.I., 1960.

The author presents an alternative proof for a theorem of König [Math. Ann. **77** (1916), 453–465]. Since the proof is constructive, it is suggested that it should lead to at least one, and perhaps several, computationally feasible algorithms for the solution of the distribution problem (sometimes called the transportation problem). The algorithm developed in the proof of the theorem does not yield a solution of the distribution problem directly. By means of row and column permutations, it reduces the cost matrix to a form which is then easily attacked by the Hungarian method of H. W. Kuhn [Naval Res. Logist. Quart. **2** (1955), 83–97; MR **17**, 759].

B. A. Galler (Ann Arbor, Mich.)

5489:

Gale, David. Transient flows in networks. Michigan Math. J. **6** (1959), 59–63.

The concept of a dynamic flow in a network was introduced by Ford and Fulkerson [Operations Res. **6** (1958), 419–433; MR **20** #1403]. A dynamic network consists of a graph Γ to each edge e of which corresponds a nonnegative integer $\gamma(e)$, called the capacity of the edge, and a second nonnegative integer $\tau(e)$, called the transit time of the edge. In terms of transportation networks, the capacity γ is to be thought of as giving an upper bound to the amount that can be shipped along an edge e, while the transit time τ specifies how long it takes a shipment to traverse this edge. In this framework, Ford and Fulkerson have considered the following problem: For a dynamic network Γ with two distinguished terminals s and s' (called the source and the sink, respectively), to determine the maximum amount μ_k that can be shipped from s to s' in k time periods. In the work referred to, the authors describe an ingenious algorithm for obtaining μ_k for each integer k. More precisely, they show, for each integer k, how to obtain a flow ϕ_k (to be thought of as a shipping schedule) that achieves the desired shipment μ_k from s to s'.

Concerning the solution of Ford and Fulkerson, the following observation may be made. In order to achieve the maximum numbers $\mu_1, \mu_2, \cdots, \mu_k$, the authors construct a sequence of flows $\phi_1, \phi_2, \cdots, \phi_k$. It would be computationally advantageous if it turned out that ϕ_2 is a "continuation" of ϕ_1 and, in general, ϕ_{i+1} a continuation of ϕ_i. Put another way, one might hope that the flow ϕ_k has the property that for each time $i<k$ the amount already shipped into s' is the maximum μ_i. In this case the single flow ϕ_k would provide a solution to the maximum problem, not only for k time periods, but also for any smaller number of periods. However, the flows obtained by the authors do not have this desirable property; indeed, it is not clear from their work that such universal maximal flows exist.

The present paper establishes the existence of universal maximal flows, not only for the case treated by Ford and Fulkerson, but also for the considerably more general case in which the capacities γ and transit times τ may vary with time.

H. W. Kuhn (Princeton, N.J.)

5490:

Gomory, Ralph E. Solving linear programming problems in integers. Proc. Sympos. Appl. Math., Vol. 10, pp. 211–215. American Mathematical Society, Providence, R.I., 1960.

Let the standard simplex form of a linear programming problem be as follows. Objective function: $z=a_{00}+\sum_{i=1}^n a_{0i}(-t_i)$; $x_j=a_{j0}+\sum_{i=1}^n a_{ji}(-t_i)$ $(j=1, 2, \cdots, m)$. Let some of the a_{j0} not be integers. Denote by f_{rs} the fractional part of a_{rs}. Then new restrictions are imposed, e.g., $s_k=-f_{k0}-\sum_{j=1}^n f_{kj}(-t_j)$. This procedure is continued until integer solutions are achieved. The total number of new restrictions cannot exceed $m+n+2$. The method is illustrated with the help of a worked example.

G. Tintner (Ames, Iowa)

5491:

Dantzig, G. B. Note on solving linear programs in integers. Naval Res. Logist. Quart. **6** (1959), 75–76.

This note presents an extremely simple method for adding linear inequality constraints to a linear programming problem so as to exclude certain nonintegral feasible solutions. Precisely, if a linear program in variables $x_1, x_2, \cdots, x_n$ has a basic feasible solution for basic variables $x_1, \cdots, x_m$, say, which is inadmissible for all admissible solutions with integral values and not the basic solution. It is not known whether these additional constraints will lead to a solution in a finite number of iterations.

H. W. Kuhn (Princeton, N.J.)

5492:

Dorn, W. S. A duality theorem for convex programs. IBM J. Res. Develop. **4** (1960), 407–413.

The paper gives a duality theorem for the problem of minimizing a convex differentiable function subject to linear constraints, namely, let x be a non-negative n-vector satisfying (1) $Ax \geq b$, and let $f(x)$ be convex and

differentiable in the region determined by (1). Minimum Problem: Minimize $f(x)$ subject to (1). Let ∇f be the gradient of f, and let u and v satisfy (2) $v \geqq 0$, $A'v + \nabla f(u) \leqq 0$, and suppose f is convex in the region determined by (2). Maximum Problem: Maximize $f(u) - u'\nabla f(u) + b'v$ subject to (2) (primes denote transposition and juxtaposition scalar product). The author states the Duality Theorem to the effect that if one of the two extremum problems has a solution, so does the other, and the values are equal. However, for part of his proof he seems to require that ∇f be invertible and that its inverse be differentiable, conditions which do not appear in his hypotheses. *D. Gale* (Providence, R.I.)

5493:

Yanai, Hiroshi. A Min Max solution of an inventory problem. Proc. Fac. Engrg. Keio Univ. **12** (1959), 114–119.

This paper contains a generalization, by means of including some additional costs, of a minimax solution to an inventory problem proposed by the reviewer in Arrow, Karlin and Scarf, *Studies in the mathematical theory of inventory and production* [Stanford Univ. Press, Stanford, Calif., 1958; MR **20** #767]. *H. Scarf* (Stanford, Calif.)

5494:

Ferrer Martín, Sebastián. Theory of games. Estadíst. Española No. 6 (1960), 44–54. (Spanish. English summary)

Elementary expository paper. *A. G. Azpeitia* (Providence, R.I.)

5495:

Howe, Charles W. An alternative proof of the existence of general equilibrium in a von Neumann model. Econometrica **28** (1960), 635–639.

The paper derives von Neumann's theorem from the theory of linear inequalities rather than by means of game theory [Kemeny, Morgenstern, and Thompson, Econometrica **24** (1956), 115–135; MR **18**, 266] or the theory of convex cones [Gale, *Information in games with finite resources*, Princeton Univ. Press, Princeton, N.J., 1957; MR **19**, 1025]. *D. Gale* (Providence, R.I.)

BIOLOGY AND SOCIOLOGY

See also 5478, 5482.

5496:

Stephenson, John L. Theory of transport in linear biological systems. I. Fundamental integral equation. Bull. Math. Biophys. **22** (1960), 1–17.

The author discusses conditions under which the input of a biological system $\gamma_a(t)$ is related to the output $\gamma_b(t)$ by a linear integral equation of the form

$$\gamma_b(t) = \int_0^t \gamma_a(\omega) W(t-\omega)\, d\omega,$$

where $W(t)$ is a transport function, characteristic of the system. The equation is solved for $W(t)$ in the case where both the input and the output are sums of exponentials. In that case the transport function is also a sum of exponentials. In principle, the parameters of the transport function can be determined from a series of experiments involving different inputs. Once the transport function characteristic of the system is determined, the time course of the output can be determined from that of the input or vice versa. *A. Rapoport* (Ann Arbor, Mich.)

5497:

Jaeger, J. C. Diffusion in branching regions. Proc. Cambridge Philos. Soc. **56** (1960), 55–63.

Author's summary: "The problem of diffusion from an instantaneous point source in a thin plane sheet which is intersected by one or more other thin plane sheets of the same material is considered. This problem arises in the study of the diffusion of packets of transmitter substance in synaptic clefts, and some numerical results for the amphibian neuro-muscular junction are presented. A similar discussion applies to branching rods and to cases where there is loss of diffusing substance at the surfaces of the sheets." *R. C. T. Smith* (Armidale)

5498:

Komatu, Yûsaku; Nishimiya, Han. On probabilities of non-paternity with reference to consanguinity. I, II. Proc. Japan Acad. **35** (1959), 612–615, 616–619.

The problem discussed in these papers is as follows. Subject to one of three conditions (A), (B), (C) specified below, we select from a random-mating population a mother, her child, and a putative father who is not the real father of the child. What is the chance that when the blood-groups of mother, child and putative father are examined (in just one blood group system, e.g., ABO or MN), the putative father will be able to show that he is not the real father? Condition (A) restricts the families to those in which the real father and mother have a given degree of relationship, and the putative father is unrelated to them; conditions (B), (C) are permutations of this with interchanges of mother, real and putative fathers. The problem is also discussed when the mother's blood group is not examined, but otherwise conditions are the same. *C. A. B. Smith* (London)

5499:

Rashevsky, N. The role of the individual in social dynamics. Bull. Math. Biophys. **22** (1960), 207–215.

Previous models proposed by the author for mass behavior, based on the propensity of individuals to imitate each other, have for the most part led to unstable systems, in which the entire population, except for a class of individuals immune to influence, is bound to swing to one or another form of behavior. The thresholds determining the direction of the swing were functions of parameters denoting average coefficients of influence of the passive (imitating) and the active (influence-immune) individuals, as well as the numbers of actives of the two classes exerting their influence in opposite directions. In particular, if there is a secular (historical) change in the relative numbers of the actives of the two classes, the threshold for a swing in the opposite direction in the behavior of the passives may occur at a given moment of history.

Since the threshold depends also on the coefficients of influence, it is clear that a change in this coefficient of the one or another class of actives can advance or retard the passing of the threshold. If the coefficient of influence of an active class is the average of a distribution, the appearance of an "exceptional" individual with an unusually high coefficient will change the average coefficient of his class. If this class is in the ascendancy, the impending swing in mass behavior will come sooner and vice versa.

A. Rapoport (Ann Arbor, Mich.)

5500:

Theodorescu, Radu. Nouvelle interprétation de certains modèles stochastiques pour apprendre. Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **28** (1960), 153–155.

The author points out that a generalization of the notion of a chain of infinite order (chaîne à liaisons complètes) which involves two sequences of random variables allows one to state compactly R. R. Bush and F. Mosteller's [*Stochastic models for learning*, Wiley, New York, 1955; MR **16**, 1136] definitions of experimenter-, subject-, and experimenter-subject-controlled events. A deeper use of this theory in the analysis of learning models is given by Lamperti and P. Suppes [Pacific J. Math. **9** (1959), 739–754; MR **21** #7567]. *R. D. Luce* (Philadelphia, Pa.)

INFORMATION AND COMMUNICATION THEORY

See also 5079.

5501:

Яглом, А. М. [Yaglom, A. M.]; Яглом, И. М. [Yaglom, I. M.]. Вероятность и информация. [Probability and information]. 2nd edition, revised and supplemented. Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1960. 315 pp. 5.40 r.

The second edition differs from the first [Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1957; MR **19**, 990] essentially in the addition of sections discussing the application of information-theoretic concepts to linguistics, music, television, and radio phototransmission.

A. Feinstein (Urbana, Ill.)

5502:

Ikeda, Sadao. Continuity and characterization of Shannon-Wiener information measure for continuous probability distributions. Ann. Inst. Statist. Math. Tokyo **11** (1959), 131–144.

Let p and p_i $(i=1, 2, \cdots)$ be probability density functions with respect to the measure m. Let the difference between $E =_{\mathrm{dt}} \{x : p(x) > 0\}$ and $E_i =_{\mathrm{dt}} \{x : p_i(x) > 0\}$ be of measure 0. If ess. $\sup_{x \in E} |p(x) - p_i(x)| \to 0$ $(i \to \infty)$ and $m(E) < \infty$, then

$$H(X_i) =_{\mathrm{dt}} \int p_i(x) \log p_i(x)\, dm \to H(X) =_{\mathrm{dt}} \int p(x) \log p(x)\, dm (i \to \infty).$$

A similar continuity theorem is given when $m(E) = \infty$. A characterization of H is given using such continuity conditions.

S. Sherman (Detroit, Mich.)

5503:

Watanabe, Satosi. Information-theoretical aspects of inductive and deductive inference. IBM J. Res. Develop. **4** (1960), 208–231.

Author's summary plus parenthetic comment: "By a straightforward application of Bayes' theorem of probability, the behaviour is discussed of the credibilities (inductive probabilities) of competing hypotheses as functions of an increasing body of relevant empirical data. It is shown how the effect of a priori [initial] credibilities persists in the evaluation of credibilities in general, except in the important limiting cases investigated. An 'inverse H-theorem' is mathematically demonstrated, according to which the entropy function defined in terms of the credibilities shows a net decrease in time. This decrease is not necessarily monotonous [monotonic] in an individual case, but is monotonous in the 'expected' behaviour of the inductive entropy function. [I think this means that the expected value of the function is monotonic decreasing.] Three machine-simulation experiments of inductive inference on the IBM 704 are described. The first two concern the classical problem of guessing the ratio of white and black balls in an urn. The third experiment concerns guessing a hidden pattern obeyed by a sequence of binary numbers." *I. J. Good* (Teddington)

5504:

Swanson, J. A. Physical versus logical coupling in memory systems. IBM J. Res. Develop. **4** (1960), 305–310.

Author's abstract: "A memory system consisting of bistable static dissipationless units such as ferrites, ferroelectrics, or cryotrons, is considered. For a given amount of physical material the memory capacity may be increased by using small volumes of the bistable material for each bit. If made sufficiently small, however, the individual bits will become unreliable because of the influence of thermal agitation and quantum-mechanical tunneling processes. Some unreliability can be tolerated, since it can be compensated by redundancy. The optimum size of the individual bit, for maximum information storage, is evaluated. If thermal agitation is the prime source of errors, then the optimum-sized bit involves typically less than 100 of the independent cooperating units (electron spins, dipoles, et cetera) which cause the bistability. The maximization process concerns itself only with the preservation of information and not with possible methods of access to the individual bit. In particular, the maximization process neglects complications in the coding equipment needed to read in and out of memory."

The author points out that the difficulties of addressing an element of only 100 units are extreme, so that the results are remote from practical possibility, but that they are more realistic than the results of previous discussions in which allowance had been made only for thermal noise and not for thermal fluctuations.

I. J. Good (Teddington)

SERVOMECHANISMS AND CONTROL

See also A4856, A4898.

5505:

Čzan, Sy-in. The theory of quality of non-linear control systems. Prikl. Mat. Meh. **23** (1959), 971–974 (Russian); translated as J. Appl. Math. Mech. **23**, 1387–1392.

A method is presented for investigating the response time of control systems consisting of linear controlled elements and a nonlinear controller. The rate of decay of the square of the Euclidean norm in a space of the actuating signal and the generalized coordinates of the controlled elements is used as an indication of the response time.

The controlled elements are described by differential equations of the form

$$\dot{\eta}_k = \sum_{\alpha=1}^{m} b_{k\alpha}\eta_\alpha + n_k\mu \quad (k = 1, \cdots, m),$$

where η_k are generalized coordinates, $b_{k\alpha}$ are constants, μ is a controller coordinate and n_k are controller constants. The controller is described by

$$v^2\ddot{\mu} + w\dot{\mu} + s\mu = f(\sigma), \quad \sigma = \sum_{\alpha=1}^{m} p_\alpha\eta_\alpha - r\mu,$$

where v, w, and s are known functions of μ and σ, and p_α and r are constants. $f(\sigma)$ is a nonlinear characteristic belonging to one of two classes: (1) $f(\sigma)=0$ for $|\sigma| \leqq \sigma_*$, $\sigma f(\sigma) > 0$ for $|\sigma| > \sigma_*$; (2) $[df(\sigma)/d\sigma]_{\sigma=0} \geqq h > 0$, $\phi(\sigma) = f(\sigma) - h(\sigma)$, $\sigma\phi(\sigma) > 0$ for $\sigma \neq 0$.

P. M. DeRusso (Troy, N.Y.)

5506:

Smith, Fred B. Time-optimal control of higher-order systems. IRE Trans **AC-6** (1961), 16–21.

Author's summary: "Practical extension of time-optimal control to systems of higher order than three has been limited primarily by difficulties in physically representing surfaces in a phase space of these higher dimensions. A method is presented here for obtaining the forcing function as a function of the state variables without requiring use of the phase space concept. On line solution of a set of transcendental equations is required. Results of a digital simulation of a fourth-order, real-root, single-degree-of-freedom system are presented. In a digital solution the system operates as a series of short open-loop control intervals. The effect of including derivatives of the input for prediction is shown for second-order model inputs."

5507:

Горская, Н. С. [Gorskaya, N. S.]; Крутова, И. Н. [Krutova, I. N.]; Рутковский, В. Ю. [Rutkovskiĭ, V. Yu.]. ★Динамика нелинейных сервомеханизмов [Dynamics of nonlinear servomechanisms]. Institut Avtomatiki i Telemehaniki. Izdat. Akad. Nauk SSSR, Moscow, 1959. 319 pp. 16.60 r.

This is a practical book, employing phase plane and point transformation methods along with numerical and graphical techniques, devoted to the study of non-linear servomechanisms describable by second or third order differential equations. Numerous examples of specific types of pneumatic, hydraulic or electrical servos are presented and discussed in considerable detail, making the book of considerable value to an analytical engineer.

J. F. Heyda (Cincinnati, Ohio)

5508:

Waligórski, S. On a theorem of E. N. Gilbert. J. Math. and Phys. **38** (1959/60), 206–207.

A counter example is given to theorem 2 of E. N. Gilbert, same J. **33** (1954), 57–67 [MR **15**, 1009]. In the language of the cited review, the theorem should read: Suppose $P \subset P(N)$. A frontal network exists which produces all the terminal states of P and uses only n relays if and only if there is a 1-1 isotone mapping of a subset of B^n into P.

S. Sherman (Detroit, Mich.)

5509:

Wang, Chuan-shan. Realizability of sequence tables of relay circuits. Sci. Sinica **8** (1959), 435–447.

The synthesis of a sequential relay circuit by the method of M. Gavrilov begins with the construction of a sequence table, that is, a table showing the time-sequential order of operation of all the relays in the circuit. Time is divided into discrete periods and one and only one relay is assumed to change its condition in any period. In a circuit containing a total of n relays (primary and secondary), each relay is designated by precisely one of the numbers $2^0, 2^1, \cdots, 2^{n-1}$. The number $1 \cdot 2^k$ denotes that relay 2^k is energized, while $0 \cdot 2^k$ denotes that it is not energized. The state of the entire circuit in any time period is specified by the "state number", a number of the form, $\sum_{k=0}^{n-1} \delta_k \cdot 2^k$, where $\delta_k = 0$ or 1 depending on the final state of relay 2^k in this period. With these conventions, the sequence table becomes essentially a sequence of state numbers which purportedly corresponds to the desired sequential action of the circuit.

However, the sequence table may not be realizable as a sequential relay circuit. Additional secondary (auxiliary) relays may be required. The paper under review presents a simple method for ascertaining the realizability of a given sequence table and for determining the actual number of additional secondary relays required to achieve realizability.

In the customary way, one distinguishes between "stable" and "unstable" states of the circuit, namely, a state is "stable" if a transition to another state can be effected only by changing the condition of one of the primary (i.e., input or receiving) relays; a state is "unstable" if it is not stable. Since it is assumed that only one relay changes condition in any time period, obviously there is only one possible transition from each unstable state. Similarly, if there are m primary relays, then there are precisely m possible transitions from each stable state. These two requirements define the realizability of the sequence table. If one or the other requirement is not satisfied, the sequence of state numbers will contain at least one repetition of a state number with an unallowable transition. From the number of such unallowable repetitions, the author derives the minimum number of additional secondary relays to make the sequence table realizable. Typical of the results is the following theorem: "Suppose in the sequence table the state number A repeats impermissibly p times; the state number B, q times; the state number C, r times; and so on, where $p > q, r, \cdots$. Besides, between any two A's, or two B's, or two C's $\cdots$, there exists at least one unrepeating period. Then the least and sufficient number of auxiliary relays required will be $m_0 = [\log_2 p]$."

E. K. Blum (Pacific Palisades, Calif.)

5510:

★**Proceedings of an international symposium on the theory of switching, 2–5 April 1957.** The Annals of the Computation Laboratory of Harvard University, Vols. XXIX, XXX. Harvard University Press, Cambridge, Mass., 1959. Part I: xl+305 pp.; Part II: vii+345 pp. $15.00.

These volumes represent the official proceedings of an international symposium organized for the purpose of either initiating communication or improving communication between researchers concerned with switching theory or its applications. The books contain 39 technical papers. Approximately one-half of the papers deal with purely mathematical aspects of switching theory. [See review below.] The remainder of the papers treat the physics and engineering of switching theory.

Each volume of the "Proceedings" is well edited (with only an occasional typographical error noticed by the reviewer), although an index would have been helpful to the reader. The works represent a definitive source for present trends in switching theory; indeed, an excellent source for some future trends in classical switching theory.

In a very real sense almost every paper in classical switching theory that has appeared since the symposium has been a footnote to one or more of the papers in the "Proceedings". *A. A. Mullin* (Urbana, Ill.)

5511:

van der Pol, Balth. Analytic treatment of real functions given in discrete points only. Proc. Internat. Sympos. Switching Theory 1957, Part I, 3–25. Harvard Univ. Press, Cambridge, Mass., 1959.

This paper is concerned with the application of mathematical analysis (in particular, analytic number theory) and elementary number theory for the representation of some real-valued functions as linear combinations of either impulse functions or step functions. The author makes good use of some of the results of his book, written with H. Bremmer, on transform theory [B. van der Pol and H. Bremmer, *Operational calculus based on the two-sided Laplace integral*, Cambridge University Press, 1955; MR 17, 363]. While all of the results are interesting they seem, to the reviewer, to be more closely allied with Shannonian information theory than with the main stream of switching theory. *A. A. Mullin* (Urbana, Ill.)

5512:

Roth, J. Paul. Algebraic topological methods in synthesis. Proc. Internat. Sympos. Switching Theory 1957, Part I, 57–73. Harvard Univ. Press, Cambridge, Mass., 1959.

The author motivates his study by pointing out that a satisfactory treatment of complex problems in the synthesis of switching systems is prior to an adequate automation of industry.

The paper treats the synthesis of combinational switching networks by making a many-one mapping of algebraic Boolean functions into combinatorial topological configurations (complexes). An algebra of these combinatorial configurations is developed which allows a visualization, different from a Venn diagram, of the conventional Boolean algebra. Then the author gives a topological interpretation to the problem of finding a Boolean function in normal form which is equivalent (i.e., has an identical truth-table with) a given Boolean function, but having a minimum number of literals in the sense of W. V. Quine [Amer. Math. Monthly **59** (1952), 521–531; **62** (1955), 627–631; **66** (1959), 755–760; MR **14**, 440; **17**, 814; **21** #7155]. The formulation automatically includes "don't care" conditions. Two algorithms for the solution of the topological interpretation are given.

Parts of this paper essentially coincide with another paper by the author [Trans. Amer. Math. Soc. 88 (1958), 301–326; MR **20** #3755]. *A. A. Mullin* (Urbana, Ill.)

5513:

Roth, J. Paul; Wagner, E. G. Algebraic topological methods for the synthesis of switching systems. III. Minimization of nonsingular Boolean trees. IBM J. Res. Develop. **3** (1959), 326–344.

[Part II is #5512; part I is the reference at the end of the above review.]

Let Γ be a map of a set G into the set of subsets of a finite set V. An element v of V is covered by g in G at a cost $\mu(g)$ if v is in $\Gamma(g)$. The paper develops and applies on "extraction algorithm" for finding a cover of minimum cost of a preassigned subset W of V. Basically the procedure partially orders G with respect to how much an element covers and its cost. The extremals (those maximal elements of G which cover an element of W not covered by any other element of G) are set aside and the algorithm is reapplied. If no extremal exists, one "branches" by considering two cases; one where a chosen element is in the minimal cover, the other where it is not.

The major share of the paper is devoted to the application of this algorithm to the design of switching circuits. As in earlier work of Roth, the problem of designing a switching circuit to produce a given Boolean function is stated in terms of cubical complexes. However all the necessary definitions are given in the present paper. An interesting feature of the paper is its division into two parts. The first part by Roth gives a concise abstract statement of the various aspects of the problem and its application to switching circuits. The second part by Wagner is addressed more to the engineering reader and proceeds primarily by means of detailed examples. This part includes a flow chart for the application of the algorithm as well as for the generation of the set G using the projection (and injection) operations introduced in the first part.

J. B. Giever (University Park, N.M.)

5514:

Ashenhurst, Robert L. The decomposition of switching functions. Proc. Internat. Sympos. Switching Theory 1957, Part I, 74–116. Harvard Univ. Press, Cambridge, Mass., 1959.

A matrix method is given for the detection and identification of some simple disjunctive decompositions of combinational switching circuits. The use of a matrix method for the synthesis of combinational circuits is demonstrated. A formal treatment of separable decomposition in the sense of Shannon [Bell System Tech. J. **28** (1949), 59–98; MR **10**, 671] is given. The paper shows how "complex" decompositions can be constructed from a few simple ones. It is shown how the structure of a Boolean function can be represented relative to separability. *A. A. Mullin* (Urbana, Ill.)

5515:

Kjellberg, Goran. Logical and other kinds of independence. Proc. Internat. Sympos. Switching Theory 1957, Part I, 117–124. Harvard Univ. Press, Cambridge, Mass., 1959.

Motivated, in part, by the concept of the linear independence of a non-empty subset of a finite dimensional linear space over the field of real numbers, the author considers a concept called "logical independence" which is a generalization of "linear independence". He gives some aspects of the concept that appear in the context of lattice theory, combinatorial analysis and elementary probability theory. Erratum: on page 121, the author's reference for an axiomatization of the concept of linear dependence (over a skew field) should be to, say, B. L. van der Waerden, *Algebra*, Springer, Berlin, 1955 [MR **16**, 1081]; the reference in his paper is misnumbered.

A. A. Mullin (Urbana, Ill.)

5516:

Singer, Theodore. Some uses of truth tables. Proc. Internat. Sympos. Switching Theory 1957, Part I, 125–133. Harvard Univ. Press, Cambridge, Mass., 1959.

The author gives ways to decompose, recognize and classify switching functions of four variables by means of finite non-empty sequences of matrices (charts). He gives two general theorems concerning the decomposition of switching functions and illustrates that they are quite reasonable by means of a (decomposition) chart. Erratum: on page 131, theorem 1 should read "Let . . . be a function without vacuous variables, . . ." instead of "Let . . . be a function with our vacuous variables, ..."

A. A. Mullin (Urbana, Ill.)

5517:

Burks, Arthur W. The logic of fixed and growing automata. Proc. Internat. Sympos. Switching Theory 1957, Part I, 147–188. Harvard Univ. Press, Cambridge, Mass., 1959.

This paper is concerned with fixed (finite) automata (see, e.g., the paper of M. O. Rabin and D. Scott [IBM J. Res. Develop. **3** (1959), 114–125; MR **21** #2559]), Turing-like automata, self-reproducing automata [see, e.g., the paper of J. von Neumann in *Cerebral mechanism in behavior. The Hixon symposium*, pp. 1–41, Wiley, New York, 1951; MR **13**, 586] and "growing" automata, which include the others. A number of equivalent definitions of "automata", in the above senses, are given together with some general theorems dealing with them.

The paper has the same over-all theme as a later paper by the author that appears in *Self-organizing systems* [Pergamon Press, New York, 1960; MR **22** #4570].

A. A. Mullin (Urbana, Ill.)

5518:

Muller, David E.; Bartky, W. S. A theory of asynchronous circuits. Proc. Internat. Sympos. Switching Theory 1957, Part I, 204–243. Harvard Univ. Press, Cambridge, Mass., 1959.

This paper expounds a comprehensive theory of speed-independent asynchronous sequential circuits based, largely, upon the results of lattice theory [G. Birkhoff, *Lattice theory*, Amer. Math. Soc., New York, 1948; MR **10**, 673].

The authors present 58 technical theorems, together with some examples of certain sequential circuits to further describe their theory. Some of the theorems lend themselves to implement the synthesis of circuits composed of two-state components.

A. A. Mullin (Urbana, Ill.)

5519:

Svoboda, Antonin. Some applications of contact grids. Proc. Internat. Sympos. Switching Theory 1957, Part I, 293–305. Harvard Univ. Press, Cambridge, Mass., 1959.

By means of some theorems and illustrative examples the author presents a graphical scheme to find a minimal form for Boolean functions. He uses the terminology employed by R. H. Urbano and R. K. Mueller [Trans. IRE **EC-5** (1956), 126–132]. *A. A. Mullin* (Urbana, Ill.)

5520:

Belevitch, Vitold. Some relations between the theory of contact networks and conventional network theory. Proc. Internat. Sympos. Switching Theory 1957, Part II, 3–12. Harvard Univ. Press, Cambridge, Mass., 1959.

This paper emphasizes algebraic topological relations between combinational relay circuits and lumped, linear, finite, passive electric circuit theory. The author essentially exposes, in a clear manner, some of the results of A. G. Lune [Izv. Akad. Nauk SSSR. Ser. Mat. **16** (1952), 405–436; MR **14**, 606] concerning Boolean matrices.

A. A. Mullin (Urbana, Ill.)

5521:

Semon, Warren. Matrix methods in the theory of switching. Proc. Internat. Sympos. Switching Theory 1957, Part II, 13–50. Harvard Univ. Press, Cambridge, Mass., 1959.

The author begins with an expository treatment of some aspects of the design of combinational circuits for producing specified switching functions with a minimum number of basic components (e.g., contacts or grids). This discussion includes use of minimizing charts [*Synthesis of electronic computing and control circuits*, Harvard Univ. Press, Cambridge, Mass., 1951; MR **13**, 497] and "decomposition charts" for the detection of "decomposable" switching functions; a Boolean function f of m (m is a positive integer) Boolean variables is non-trivially "decomposable" if there exists a positive integer $n<m$ and Boolean functions $F, y_1, y_2, \cdots, y_n$, where F is defined on all of the variables $y_1, \cdots, y_n$ and the y_i's are defined on variables which form non-empty disjoint subsets of $\{x_1, \cdots, x_m\}$ such that $f(x_1, x_2, \cdots, x_m) = F(y_1, y_2, \cdots, y_n)$. Then, by means of numerous examples and theorems, the author considers the analysis of connection matrices and their transformations which leave invariant the determinants representing the outputs of circuits. He gives an expository treatment of a method for the simultaneous solution of a non-empty finite set of Boolean equations. Finally he presents a sequence of theorems for a theoretical (but not necessarily practical) solution to the problem of synthesizing a minimum contact network for a prescribed switching function. *A. A. Mullin* (Urbana, Ill.)

5522:

Hohn, Franz E. $2N$-terminal contact networks. Proc. Internat. Sympos. Switching Theory 1957, Part II, 51–58. Harvard Univ. Press, Cambridge, Mass., 1959.

By means of examples and a theorem due to M. L. Cetlin [Dokl. Akad. Nauk SSSR **86** (1952), 525–528; MR **14**, 606] the author presents some uses of Boolean matrices for an analysis of some contact networks with an equal number of input and output lines and for an analysis of cascades of such networks.

Erratum: On page 56, the first equation in section 4 should have the logical sum indexed on k' rather than k.

A. A. Mullin (Urbana, Ill.)

5523:

Calingaert, Peter. Multiple-output relay switching circuits. Proc. Internat. Sympos. Switching Theory 1957, Part II, 59–73. Harvard Univ. Press, Cambridge, Mass., 1959.

The author treats multiple-output contact networks by reducing that problem to a certain single-output problem (i.e., a two-terminal contact network) whose minimum solution yields a minimum solution to the given problem. He gives an example of a circuit which can be dealt with in this fashion.

He defines "... an important class of circuits, which will be called 'A-derivable' circuits". An 'A-derivable' circuit is a combinational circuit which satisfies the conditions that (i) ideal potential sources are never short-circuited (ideal current sources are excluded here; they occur in a dual theory) and (ii) no pair of relay coils are ever placed in series. Then he defines a "minimal circuit" as an A-derivable circuit that does not have more components than any other A-derivable circuit for producing the same switching function.

He uses these two definitions together with some examples and theorems to describe his systematic "A-derivation" method of synthesis of minimal multiple-output relay switching circuits.

A. A. Mullin (Urbana, Ill.)

5524:

Povarov, Gellius N. A mathematical theory for the synthesis of contact networks with one input and k outputs. Proc. Internat. Sympos. Switching Theory 1957, Part II, 74–94. Harvard Univ. Press, Cambridge, Mass., 1959.

This comprehensive paper, well-documented with the Anglo-Soviet literature, is concerned with, among other things, a systematic study of quantitative estimates of complexity for contact networks with a single-input line and k-output lines (called "(1, k)-networks").

Put $\nu=2^n$, n a positive integer. The author sharpens Shannon's upper bound of $2\cdot 2^\nu$ [Bell System Tech. J. **28** (1949), 59–98; MR **10**, 671] for the number of contacts necessary to realize all of the 2^ν Boolean functions of n variables. He points out, rightly, a flaw in the above paper by Shannon, whereby the estimate $M+1+2^{M+1}\geq n>M+2^M$, dealing with the synthesis of a function of n variables obtained by breaking off the tree at the $(n-m)$ bay, should be $M+1+2^{M+1}>n\geq M+2^M$. However the reviewer points out that the author is mistaken, when he says, on page 78, "... and not $6\leq n\leq 13$, as stated by Shannon". Actually, on page 76 of Shannon's paper cited above, Shannon has $6<n\leq 11$ (the reviewer has added the, obvious, $6<n$).

The author uses these estimates together with various methods of synthesis (e.g., cascading of networks and decomposition of function) to detect a minimal synthesis.

Erratum: On page 76, line 5 read "$(k-n)$-sequence" for "(kmn)-sequence". *A. A. Mullin* (Urbana, Ill.)

5525:

Dunham, Bradford; North, James H. The use of multipurpose logical devices. Proc. Internat. Sympos. Switching Theory 1957, Part II, 192–200. Harvard Univ. Press, Cambridge, Mass., 1959.

This paper discusses some logico-physical aspects of switching devices, called "multipurpose bias devices" by the authors. The devices have n (n is a positive integer ≥ 2) input lines of which p $(0<p<n)$ of the input lines are "biased" (i.e., the signals on them are fixed to be either always "on" or always "off"). There exist devices for which the biases can be adjusted so that the device realizes various logical compositions or a singulary operation on the remaining $(n-p)$ variables. To this extent multipurpose bias devices resemble neuron-like components. The authors show how a full adder, which records the mod 2 sum and "carry" on three binary input variables, can be used to yield "exclusive-or", "and", "or", "if and only if" and "not" by putting the appropriate bias signal on the appropriate input lines.

A. A. Mullin (Urbana, Ill.)

5526:

van Wijngaarden, A. The state of computer circuits containing memory elements. Proc. Internat. Sympos. Switching Theory 1957, Part II, 213–224. Harvard Univ. Press, Cambridge, Mass., 1959.

The author deals with a mathematical investigation of switching circuits containing memory devices.

He says, "Let the memory elements be numbered 1, 2, $\cdots$ and let a_r be the digit stored in the rth memory element. Then one can form a number $\sum a_r m^r$ ... Conversely, this number determines the state of the box uniquely ... The state A_n can be represented as a number, but this form is too vague and perhaps too clumsy." The reviewer points out that such a characterization can, indeed, be inconvenient in that the theoretical, somewhat interesting, case of a recursively enumerable set of memory devices cannot be so represented unless the devices are indexed on the negative integers. In this case assuming the a_i to be non-negative integers, one can even obtain a unique representation of the state of the circuit by letting m be a negative integer less than -1 (thereby, also, eliminating the useless cases of $m=0$ and $m=1$ and the oscillating case with $m=-1$).

The author gives a theorem concerning "linear boxes", e.g., one-digit delay lines and two-digit serial adders. He discusses coders (i.e., linear boxes which transform sequences of digits) and switching circuits in which the output signal is fed back to all of the input lines.

A. A. Mullin (Urbana, Ill.)

5527:

Seshu, Sundaram; Hohn, Franz E. Symmetric polynomials in Boolean algebras. Proc. Internat. Sympos. Switching Theory 1957, Part II, 225–234. Harvard Univ. Press, Cambridge, Mass., 1959.

The authors make some uses of theorems due to M. H. Stone (e.g., his Representation Theorem) [Proc. Nat. Acad. Sci. U.S.A. **20** (1935), 103–105; Trans. Amer. Math. Soc. **40** (1936), 37–111] to treat certain two-sided modules

of symmetric polynomials over a Boolean algebra. They show, among other things, that the module of symmetric polynomials over a finite Boolean ring B has $(n+1)$ dimensions, where n is the number of variables. No practical application of the theorems is conjectured.

A. A. Mullin (Urbana, Ill.)

5528:

Holbrook, B. D. Some logical requirements for the control of switching networks. Proc. Internat. Sympos. Switching Theory 1957, Part II, 235–240. Harvard Univ. Press, Cambridge, Mass., 1959.

By means of elementary examples the author gives "a brief examination . . . of the basic objectives of the central office designer and some of the practical limitation within which he must work . . .".

A. A. Mullin (Urbana, Ill.)

5529:

Rubinoff, Morris. Remarks on the design of sequential circuits. Proc. Internat. Sympos. Switching Theory 1957, Part II, 241–280. Harvard Univ. Press, Cambridge, Mass., 1959.

This paper, flavored with a military-like terminology, represents a comprehensive summary, in suggestive language, of papers on the theory of sequential circuits by E. F. Moore [*Automata studies*, pp. 129–153, Princeton Univ. Press, Princeton, N.J., 1956; MR **17**, 1140], G. H. Mealy [Bell System Tech. J. **34** (1955), 1045–1079; MR **17**, 436] and D. A. Huffman [J. Franklin Inst. **257** (1954), 161–190, 275–303; MR **15**, 1009].

He introduces the concepts of a "command graph" and "dispatcher tables". Roughly speaking, the former concept corresponds to a state transition diagram, whose nodes have been modified to accommodate both the present state of the circuit and the present input signal, and the latter concept, which is obtained from the former one, corresponds to Huffman's "flow table".

The author demonstrates, by means of examples, how to use certain matrix methods to go from a dispatcher table to a circuit.

He shows with examples, how the method permits the detection of hazards. *A. A. Mullin* (Urbana, Ill.)

5530:

Moore, Edward F. The shortest path through a maze. Proc. Internat. Sympos. Switching Theory 1957, Part II, 285–292. Harvard Univ. Press, Cambridge, Mass., 1959.

This paper is concerned with some aspects of operations research as applied to a study of "mazes". A "maze" is an entity consisting of (i) a finite set of at least two nodes, (ii) two distinguished nodes, one called the "start" and the other called the "finish" and (iii) a set of (possibly directed or weighed) edges joining the nodes (at most one edge between any two nodes) so that there is at least one path from the "start" to the "finish".

The author gives three algorithms for finding all of the minimal paths from the "start" to the "finish" of a maze; a unique minimal path is called "the shortest path". He, also, gives an algorithm for finding a minimum "cost" route through a maze whose edges are weighted; the reviewer supposes that a unique minimum cost route might be called "the smallest cost" route.

Two of the algorithms are suited for synchronous circuit operation and the other two algorithms are suited for either synchronous or asynchronous circuit operation.

For additional reading along these lines the reader should consult the paper by R. Bellman in Quart. Appl. Math. **16** (1958), 87–90 [MR **21** #1228].

A. A. Mullin (Urbana, Ill.)

5531:

Walther, Alwin. Switching research in Germany. Proc. Internat. Sympos. Switching Theory 1957, Part II, 295–301. Harvard Univ. Press, Cambridge, Mass., 1959.

Three large German companies and four German universities and technical institutes have built electronic computers for either industrial or scientific use. Economic and investigative reasons have prompted research in switching theory. *A. A. Mullin* (Urbana, Ill.)

5532:

Vetuhnovskiĭ, F. Ya. On the number of indecomposable nets and some of their properties. Dokl. Akad. Nauk SSSR **123** (1958), 391–394. (Russian)

A subnet of a net S is said to be nontrivial if it consists of more than a single vertex and does not coincide with S; a net is said to be indecomposable if it has no nontrivial subnets. The author furnishes two sets of necessary and sufficient conditions that a net be indecomposable. In order to obtain effectively a lower bound for the number of indecomposable nets with n vertices, a class of non-isomorphic indecomposable nets is constructed effectively, and it is shown that for n sufficiently large the number of elements in the set having n vertices is greater than $\{n/(2e \ln^2 n)\}^n$. *E. J. Cogan* (Bronxville, N.Y.)

This paper is concerned with some aspects of switching [illegible]

[illegible]

of symmetric polynomials over a Boolean ring. They also [illegible] that the inversion of symmetric polynomials over a field. Theorem may be [illegible] dimensional, where n is the number of variables. No practical application of the theorem is contemplated. [illegible]

[illegible]

Holbrook, R. D. Some logical requirements for the [illegible] Proc. Internat. Sympos. [illegible] Harvard Univ. Press, Cambridge, Mass. [illegible]

[illegible]

Huffman, David A. The synthesis of sequential switching circuits. [illegible] Part II [illegible]

This paper [illegible]

[illegible]

Moore, Edward F. [illegible] Proc. Internat. Sympos. [illegible] Harvard Univ. Press, Cambridge, Mass. [illegible]

[illegible]

AUTHOR INDEX

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